

AMIETE – CS (Current & New Scheme)

Time: 3 Hours

December - 2017

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

- a. Let S and T be language over $\Sigma = \{a,b\}$ represented by the regular expressions $(a+b)^*$ and $(a+b)^*$, respectively. Which of the following is true?

- (A) $S \subset T$ (B) $T \subset S$
(C) $S = T$ (D) $S \cap T = \emptyset$

- b. Assume the statements S1 and S2 given as:

S1: Given a context free grammar G, there exist an algorithm for determining whether $L(G)$ is infinite.

S2: There exists an algorithm to determine whether two context free grammars generate the same language.

Which of the following is true?

- (A) S1 is correct and S2 is not correct
(B) Both S1 and S2 are correct
(C) Both S1 and S2 are not correct
(D) S1 is not correct and S2 is correct

- c. Regular expression for the language $L = \{ w \in \{0, 1\}^* \mid w \text{ has no pair of consecutive zeros} \}$ is

- (A) $(1 + 010)^*$ (B) $(01 + 10)^*$
(C) $(1 + 010)^* (0 + \lambda)$ (D) $(1 + 01)^* (0 + \lambda)$

- d. Which is not the correct statement?

- (A) The class of regular sets is closed under homeomorphisms.
(B) The class of regular sets is not closed under inverse homeomorphisms.
(C) The class of regular sets is closed under quotient
(D) The class of regular sets is closed under substitution.

- e. Consider the following statements:

I. Recursive languages are closed under complementation.

II. Recursively enumerable languages are closed under union.

III. Recursively enumerable languages are closed under complementation.

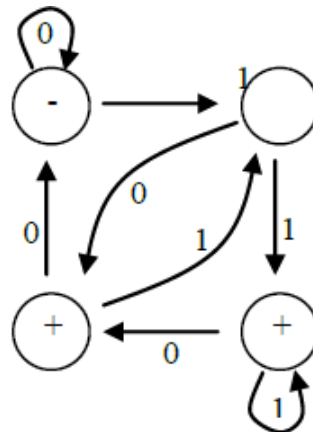
Which of the above statements are true ?

- (A) I only (B) I and II
(C) I and III (D) II and III

- f. Given $L_1=L(a^*baa^*)$ and $L_2=L(ab^*)$. The regular expression corresponding to language $L_3 = L_1/L_2$ (right quotient) is given by
 (A) a^*b (B) a^*baa^*
 (C) a^*ba^* (D) None of these
- g. $S \rightarrow aSa \mid bSb \mid a \mid b$; The language generated by the above grammar over the alphabet $\{a, b\}$ is the set of
 (A) All palindromes
 (B) All odd length palindromes
 (C) Strings that begin and end with the same symbol
 (D) All even length palindromes
- h. A minimum state deterministic finite automation accepting the language $L = \{W \mid W \in \{0,1\}^*, \text{ number of 0's and 1's in } W \text{ are divisible by 3 and 5 respectively}\}$ has
 (A) 15 States (B) 11 States
 (C) 10 States (D) 9 States
- i. Which of the following is TRUE?
 (A) Every subset of a regular set is regular
 (B) Every finite subset of a non-regular set is regular
 (C) The union of two non-regular sets is not regular
 (D) Infinite union of finite sets is regular
- j. For $S \in (0+1)^*$ let $d(s)$ denotes the decimal value of s (e.g., $d(101)= 5$).
 Let $L = \{s \in (0 + 1)^* \mid d(s) \bmod 5 = 2 \text{ and } d(s) \bmod 7 \neq 4\}$
 Which one of the following statements is true?
 (A) L is recursively enumerable, but not recursive
 (B) L is recursive, but not context-free
 (C) L is context-free, but not regular
 (D) L is regular

**Answer any FIVE Questions out of EIGHT Questions.
 Each question carries 16 marks.**

- Q.2** a. Design a DFA (*deterministic* finite automaton) to accept the language $L_1 = \{\alpha \in \{a, b, c\}^* \mid \alpha \text{ starts and ends with the same symbol}\}$. Draw the transition diagram of DFA, and clearly indicate the start state and the final state(s). (10)
- b. Draw the corresponding NFA to the following DFA. (6)



- Q.3** a. Let, L_2 denotes the context-free language, $\{\alpha\alpha^R \mid \alpha \in \{a, b\}^*\}$ where, α^R stands for the reverse of the string α . Prove or disprove: The complement of L_2 (that is, $\sim L_2 = \{a, b\}^* \setminus L_2$) is context-free. Either construct a CFG/PDA to accept $\sim L_2$, or supply a proof based on the pumping lemma. (8)
- b. Let $\Sigma = \{a, b\}$; find a grammar that generates the following language. (8)
 (i) $L = \{a^n b^m; n \geq 0, m > n\}$
 (ii) $L = \{a^n b^{n-3}; n \geq 3\}$
- Q.4** a. Construct the right and left linear grammar for the language: (8)
 $L = \{a^n b^m; n \geq 2, m \geq 3\}$
- b. Find the language for the given regular expression: $r = (aa)^* (bb)^* b$ (4)
- c. Find the regular expression for the language $L(r) = \{w \in \{0,1\}^*; w \text{ has at least one pair of consecutive zeros.}\}$ (4)
- Q.5** a. Let the language, $L = \{a^i b^j c^k \mid i \geq 1\}$ is given. Show that L_1 is not a context free language using pumping lemma. (10)
- b. Explain the undecidability of Post's correspondence problem. (6)
- Q.6** a. Give a CFG to generate $A = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and either } i = j \text{ or } j = k\}$. Is the grammar ambiguous? Why or why not? (6)
- b. Convert the following CFG into Chomsky normal form. (10)
 $A \rightarrow BAB \mid B \mid \epsilon$
 $B \rightarrow 00 \mid \epsilon$
- Q.7** a. Convert the CFG G given below to an equivalent PDA. (8)
 $E \rightarrow E + T \mid T$
 $T \rightarrow T \times F \mid F$
 $F \rightarrow (E) \mid a$
- b. Construct a PDA for the language of all non-palindromes over $\{a, b\}$. (8)
- Q.8** a. Design Turing machines for the language $L = \{a^n b^n c^n \mid n \geq 1\}$ (6)
- b. Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free. Then use the above to show that the language given below is not a CFL. $A = \{w \mid w \in \{a, b, c\}^* \text{ and contains equal numbers of } a\text{'s, } b\text{'s and } c\text{'s}\}$. (10)
- Q.9** a. Give the proof that a language is decidable if and only if it is Turing recognizable and co-Turing recognizable. (8)
- b. Prove that; A Turing Machine = $\langle M, w \rangle$: M is a TM and M accepts w is not a decidable language. (8)