ROLL NO. ___________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **December - 2017** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Seven distinct car accidents occurred in a week. What is the probability that they all occurred on the same day?

(A) $1/7^7$ **(B)** $1/7^6$
(C) $1/2^7$ **(D)** $7/2^7$

b. Identify the correct translation into logical notation of the following assertion. Some boys in the class are taller than all the girls.
Note: taller (x,y) is true if x is taller than y.

(A) $(\exists x)\ (boy(x) \rightarrow (\forall y)\ (girl(y) \wedge taller(x,y)))$
(B) $(\exists x)\ (boy(x) \wedge (\forall y)\ (girl(y) \rightarrow taller(x,y)))$
(C) $(\exists x)\ (boy(x) \rightarrow (\forall y)\ (girl(y) \rightarrow taller(x,y)))$
(D) $(\exists x)\ (boy(x) \wedge (\forall y)\ (girl(y) \wedge taller(x,y)))$

c. Let A, B and C be non-empty sets and let X= (A–B)-C and Y= (A–C)-(B–C). Which one of the following is TRUE?

(A) $X=Y$ **(B)** $X\subset Y$
(C) $Y\subset X$ **(D)** None of these

d. Consider the following propositional statements:
P1: $((A\wedge B) \rightarrow C)) \equiv ((A \rightarrow C) \wedge (B \rightarrow C))$
P2: $((A\vee B) \rightarrow C)) \equiv ((A \rightarrow C) \vee (B \rightarrow C))$
Which one of the following is true?

(A) P1 is a tautology, but not P2
(B) P2 is a tautology, but not P1
(C) P1 and P2 are both tautologies
(D) Both P1 and P2 are not tautologies

e. The solution to the recurrence equation $T(2^k) = 3T(2^{k-1}) +1$, $T(1)=1$ is

(A) 2^k **(B)** $(3^{k+1}-1)/2$
(C) $3^{\log_2 k}$ **(D)** $2^{\log_3 k}$

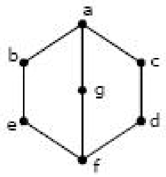
f. "If X then Y unless Z" is represented by which of the following formulae in propositional logic?

(A) $(X\wedge\neg Z)\rightarrow Y$ **(B)** $(X\wedge Y)\rightarrow \neg Z$
(C) $X\rightarrow(Y\wedge \neg Z)$ **(D)** $(X\rightarrow Y)\wedge \neg Z$

g. The set {1,2,3,5,7,8,9} under multiplication modulo 10 is not a group. Given below are four plausible reasons. Which one of them is false?
(A) It is not closed **(B)** 2 does not have an inverse
(C) 3 does not have an inverse **(D)** 8 does not have an inverse

h. Consider (Z,*) be an algebraic structure, where Z is the set of integers and the operation * is defined by n*m=maximum(n,m). Which of the following statements is true for (Z, *)?
(A) (Z, *) is a monoid **(B)** (Z, *) is an abelian group
(C) (Z, *) is a group **(D)** None of these

i. In the lattice defined by the Hasse diagram given in the following figure, how many complements does the element 'e' have?



(A) 2
(B) 3
(C) 0
(D) 1

j. Let f: A→B be an injective (one to one) function. Define g: $2^A \to 2^B$ as: g(C) = {f(x) | x ϵ C}, for all subsets C of A. Define h: $2^B \to 2^A$ as: h(D)= {x | x ϵ A, f(x) ϵ D}, for all subsets D of B.Which of the following statements is always true?
(A) g(h(D)) ⊆ D **(B)** g(h(D)) ⊇ D
(C) g(h(D)) ⋂D= Φ **(D)** g(h(D)) ⋂(B-D)≠Φ

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 **a**. If U= {1,2,3,4,5,6,7,8,9}, A={1,2,4,6,8}, and B= {2,4,5,9}, compute the following **(4)**
(i) $\overline{A}$ (ii) $\overline{B}$ (iii) $\overline{A} \cup \overline{B}$ (iv) $\overline{A \cup B}$
(v) $\overline{A} \cap \overline{B}$ (vi) $\overline{A \cap B}$ (vii) B-A (viii) A-B

b. A survey of a sample of 25 new cars being sold by an auto-dealer was conducted to see which of the three popular options: air-conditioning, radio and power windows, were already installed. The survey found: 15 had air-conditioning, 12 had radio, 11 had power windows, 5 had air- conditioning and power windows, 9 had air-conditioning and radio, 4 had radio and power windows, and 3 had all the three options. Find the number of cars that had
(i) Only power windows (ii) Only air-conditioning
(iii) Only radio (iv) Only one of the options
(v) At-least one option (vi) None of the options **(12)**

Q.3 a. Prove that, for any propositions p, q, r, the compound proposition
[(p→q) ∧ (q→r)] → (p→r) is a tautology. **(8)**

b. Prove the following logical equivalences without using truth table(s):
(i) p∨[p∧(p∨q)] ⇔ p (ii) [p∨q∨(¬p∧¬q∧ r)] ⇔ (p∨q∨r)
(iii) [(¬p∨¬q) →(p∧q∧r)] ⇔ p∧q (iv) [(p∨q)∧(p∨¬q)]∨q ⇔ p∨q **(8)**

Q.4 a. Test whether the following arguments are valid:
(i) p→q, r→s, p∨r. Hence q∨s
(ii) (¬p∨q)→r, r→(s∨t), ¬s∧¬u, ¬u→¬t. Hence p **(8)**

b. (i) Write the following proposition in symbolic form and find its negation: "All integers are rational numbers and some rational numbers are not integers"

(ii) Write the following proposition in symbolic form and find its negation: "If all triangles are right-angled, then no triangle is equiangular"
(iii) Let n be an integer. Prove that if n^2 is odd, then n is odd by indirect method of proof. **(8)**

Q.5 a. Prove by mathematical induction that, for every positive integer n, 5 divides n^5-n. **(8)**

b. If F_i's are the Fibonacci numbers and L_i's are the Lucas numbers, prove that $L_n= F_{n-1}+ F_{n+1}$ **(8)**

Q.6 a. For any non-empty sets A,B,C, prove that $A\times (B\cap C) = (A\times B) \cap (A\times C)$ **(3)**

b. Let A= {1,2,3,4,5}. Define a relation R on A×A by $(x_1,y_1)R(x_2,y_2)$ if $x_1+y_1=x_2+y_2$
(i) Verify that R is an equivalence relation on A×A.
(ii) Determine the equivalence classes [(1,3)], [(2,4)] and [(1,1)].
(iii) Determine the partition of A×A induced by R. **(12)**

c. Let R be a relation on a set A. Prove that R is reflexive if $\overline{R}$ is irreflexive. **(1)**

Q.7 a. Let f: R→R be defined by

$$f(x)=\begin{cases} 3x-5 & \text{for } x>0 \\ -3x+1 & \text{for } x\le 0 \end{cases}$$

Find $f^{-1}(0)$, $f^{-1}(1)$, $f^{-1}(-1)$, $f^{-1}(3)$, $f^{-1}(-3)$, $f^{-1}(-6)$. **(3)**

b. The functions f: R→R and g: R→R are defined by f(x)= 3x+7 for all x∈R, and $g(x)= x(x^3-1)$ for all x∈R. Verify that f is one-to-one but g is not. **(3)**

c. Let f: A→B, g: B→C, h: C→D be three functions. Prove that $(h\circ g)\circ f= h\circ(g\circ f)$ **(5)**

d. If f: A→B and g: B→C are invertible functions, then $g\circ f$: A→C is an invertible function and $(g\circ f)^{-1}= f^{-1}\circ g^{-1}$ **(5)**

Q.8 a. Let G be the set of all non-zero real numbers and let a * b= ½(ab). Show that (G,*) is an abelian group. **(8)**

b. Consider the symmetric group S_3 and the sub group H= $\{p_0, p_5\}$ thereof. Find all the right cosets of H in S_3 and hence obtain a right coset decomposition of S_3. **(4)**

c. For a group G, prove that the function f: G→G defined by $f(a)=a^{-1}$ is an isomorphism if G is abelian **(4)**

Q.9 a. For the encoding function, $E: Z^2{}_2 \rightarrow Z^5{}_2$ defined by E(00)=00001, E(01)= 01010, E(10)= 10100, E(11)=11111, find the minimum distance between the code words. Indicate the error-detecting and error-correcting capabilities of each code. **(2)**

b. The generator matrix for an encoding function $E: Z^3{}_2 \rightarrow Z^6{}_2$ is given by

$$G = \begin{matrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{matrix}$$

(i) Find the code words assigned to 110 and 010
(ii) Obtain the associated parity-check matrix
(iii) Decode the received word: 110110
(iv) Show that the decoding of 111111 is not possible by using H **(8)**

c. If the elements of a ring R form a cyclic group under addition, prove that R is commutative **(3)**

d. Prove that every finite integral domain is a field **(3)**