

DiplETE - ET/CS

Time: 3 Hours

DECEMBER 2014

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. The value of the $\lim_{x \rightarrow 0} \frac{\tan ax}{\tan bx}$ is

- | | |
|-------------------|-------------------|
| (A) $\frac{b}{x}$ | (B) $\frac{a}{x}$ |
| (C) $\frac{a}{b}$ | (D) a.b |

b. The value of $\int_0^{\pi/2} \sin^5 x \cdot \sin^7 x dx$ is

- | | |
|---------------------------|---------------------------|
| (A) $\frac{230\pi}{2045}$ | (B) $\frac{231\pi}{2048}$ |
| (C) $\frac{232\pi}{2048}$ | (D) $\frac{233\pi}{2040}$ |

c. The value of $\frac{(1+i)(2+i)}{3+i}$ in the form of $(a + ib)$ is equal to

- | | |
|----------------------------------|----------------------------------|
| (A) $\frac{2}{3} + \frac{1}{2}i$ | (B) $\frac{2}{3} - \frac{1}{2}i$ |
| (C) $\frac{3}{4} - \frac{4}{5}i$ | (D) $\frac{3}{5} + \frac{4}{5}i$ |

d. If $\vec{a} = i + 3j - 2k$ and $\vec{b} = i + 3k$, then $|\vec{a} \times \vec{b}|$ is

- | | |
|-----------------|-----------------|
| (A) $\sqrt{91}$ | (B) $\sqrt{90}$ |
| (C) $\sqrt{75}$ | (D) $\sqrt{76}$ |

e. The value of λ for which the vectors $\vec{a} = 3i + 2j + 9k$ and $\vec{b} = i + \lambda j + 3k$ are parallel to each other is equal to

- (A) -2/3 (B) 1/2
(C) -1/2 (D) 2/3

f. If $\frac{d^2y}{dx^2} - 4y = x^2$, then C.F. is

- (A) $e^{2x}(c_2 + c_1)$ (B) $e^{-2x}(c_1 + c_2)$
(C) $c_1 e^{2x} + c_2 e^{-2x}$ (D) $c_1 e^x + c_2 e^{-x}$

g. The fourier coefficient of $\int_{-\pi}^{\pi} \sin mx \cdot \cos nx \, dx$ is equal to

- (A) 0 (B) 1
(C) 2 (D) -1

h. The value of $L [t \sin 2t]$ is equal to

- (A) $\frac{4s}{s^2 + 4}$ (B) $\frac{4s}{(s^2 - 4)^2}$
(C) $\frac{4s}{(s^2 + 4)^2}$ (D) $\frac{4s}{s^2 - 4}$

i. The value of $L \{t^3 e^{-2t}\}$ is equal to

- (A) $\frac{6}{(s+2)^2}$ (B) $\frac{6}{(s-2)^2}$
(C) $\frac{6}{(s-2)^4}$ (D) $\frac{6}{(s+2)^4}$

j. The value of $L^{-1}\left\{\frac{s}{(s^2 + 1)^2}\right\}$ is equal to

- (A) $\frac{1}{2} \sin ht$ (B) $\frac{t}{2} \sin ht$
(C) $\frac{t}{2} \cos ht$ (D) $-\frac{1}{2} \sin ht$

**Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.**

Q.2 a. Evaluate $\lim_{x \rightarrow 0} \left(\frac{\cos Ax - \cos Bx}{x^2} \right)$ **(8)**

b. Expand e^x in powers of $(x + 3)$. **(8)**

Q.3 a. Evaluate $\int_0^2 \sqrt{\frac{2+x}{2-x}} dx$ **(8)**

b. Find the length of the curve $x = a \cos^3 \theta, y = a \sin^3 \theta$, in the first quadrant. **(8)**

Q.4 a. Separate into real and imaginary part of $\sec(x + iy)$ **(8)**

b. If two impedance $Z_1 = 50 \angle -40^\circ$ and $Z_2 = 70 \angle 30^\circ$ are connected in series, find the total impedance in polar form. **(8)**

Q.5 a. If $\vec{a} = i + 2j - 3k$ and $\vec{b} = 3i - j + 2k$, show that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other. Also find the angle between $2\vec{a} + \vec{b}$ and $\vec{a} + 2\vec{b}$. **(8)**

b. A rigid body is spinning with an angular velocity of 27 radian / sec about an axis parallel to $2\hat{i} + \hat{j} - 2\hat{k}$ passing through the point $\hat{i} + 3\hat{j} - \hat{k}$. Find the velocity of the point whose position vector is $4i + 8j + k$. **(8)**

Q.6 a. Solve the equation $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + y = x e^x \cdot \sin x$ **(8)**

b. The deflection of a strut of length ℓ with one end ($x = 0$) build-in and the other supported and subjected to end thrust P, satisfies the equation

$$\frac{d^2 y}{dx^2} + a^2 y = \frac{a^2 R}{P} (\ell - x)$$

Prove that the deflection curve is $y = \frac{R}{P} \left(\frac{\sin ax}{a} - \ell \cos ax + \ell - x \right)$, where

$$a\ell = \tan a\ell \quad \text{--- (8)}$$

Q.7 a. Obtain the Fourier series to represent the function $f(x) = |x|$ for $-\pi < x < \pi$

and hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots$ **(8)**

- b. Find the Fourier series for the function $f(x) = x$ in the interval $[-\pi, \pi]$ (8)
- Q.8** a. Find the Laplace transform of $e^{-t}(3 \sin t - \cos^2 t)$ (8)
- b. Find the Laplace transform of $t \sin^2 t$ (8)
- Q.9** a. Find $L^{-1} \left\{ \frac{2s^2 - 4}{(s-2)(s+\ell)(s-3)} \right\}$ (8)
- b. Find $L^{-1} \left\{ \frac{\ell}{s^3(s^2 + \ell)} \right\}$ (8)