

DiplETE – ET/CS {NEW SCHEME}

Time: 3 Hours

DECEMBER 2014

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the limit $\lim_{x \rightarrow 0} \frac{\cos ax - \cos bx}{x^2}$ is

(A) $\frac{a^2 + b^2}{2}$

(B) $\frac{b^2 - a^2}{3}$

(C) $\frac{b^2 - a^2}{2}$

(D) $\frac{a^2 - b^2}{2}$

b. The value of $\int_0^{\pi/2} \sin^5 x \cdot \cos^3 x dx$ is equal to

(A) $-\frac{1}{24}$

(B) $\frac{1}{24}$

(C) $\frac{1}{12}$

(D) $-\frac{1}{12}$

c. The complementary function of the equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$ is equal to

(A) $y = c_1 e^{3x} + c_2 e^{5x}$

(B) $y = c_1 e^{-3x} + c_2 e^{-5x}$

(C) $y = (c_1 + c_2) e^{3x}$

(D) $y = (c_1 + c_2) e^{5x}$

- d. If $z = 1 + i\sqrt{3}$, then the value of $z^2 + 4$ is equal to
- (A) $3z$ (B) $4z$
(C) z (D) $2z$
- e. The expression of $6e^{\frac{5\pi}{6}i}$ in the form of $(a + ib)$ is equal to
- (A) $-3\sqrt{3} + 3i$ (B) $3\sqrt{3} + 3i$
(C) $3\sqrt{3} + i$ (D) $3\sqrt{3} - 3i$
- f. The inverse Laplace Transform of $\tan^{-1}\frac{1}{s}$ is equal to
- (A) $\sin t$ (B) $\frac{\cos t}{t}$
(C) $\frac{\sin t}{t}$ (D) $\cos t$
- g. Laplace transform of $\cos^2 t$ is equal to
- (A) $\frac{1}{2}\left[\frac{s}{s^2+4} - \frac{1}{s}\right]$ (B) $\frac{1}{2}\left[\frac{s}{s^2+4} + \frac{1}{s}\right]$
(C) $\left[\frac{s}{s^2+4} + \frac{1}{s}\right]$ (D) $\left[\frac{s}{s^2+4} - \frac{1}{s}\right]$
- h. The period of the function of $\tan \pi x$ is equal to
- (A) ∞ (B) 0
(C) -1 (D) 1
- i. If \vec{a} and \vec{b} are two vectors such that $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 3$, then the angle between the vectors is equal to
- (A) 60° (B) 90°
(C) 30° (D) 45°
- j. The current in a circuit is $10 - 2j$ amperes when the voltage across the circuit is $60 + 20j$ volts, then the magnitude of admittance is equal to
- (A) $\left(\frac{9}{50} + \frac{1}{25}j\right)$ (B) $\left(\frac{9}{50} - \frac{1}{25}j\right)$
(C) $\left(\frac{7}{50} - \frac{2}{25}j\right)$ (D) $\left(\frac{7}{50} + \frac{2}{25}j\right)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{x}$ (8)

b. Show that the function $f(x) = |x|$ is not differentiable at $x = 0$ (8)

Q.3 a. Compute the arc-length of the curve $ay^2 = x^3$ from $x = 0$ to a point having $x = 5a$. (8)

b. Find the length of an arch of the cycloid whose equations are $x = a(\theta + \sin \theta)$ and $y = a(1 + \cos \theta)$ (8)

Q.4 a. Use De Moivre's Theorem to solve the equation $x^7 + x^4 + x^3 + 1 = 0$ (8)

b. Two circuits of impedance $2 + 4j$ ohms and $3 + 4j$ ohms are connected in parallel and A.C. voltage of 100 volts is applied across the parallel combination. Calculate the magnitude of the currents as well power factor for each circuit and magnitude of the total current for the parallel combination and its power factor. (8)

Q.5 a. A rigid body is rotating with angular velocity 2 radian/sec about an axis OR where R is $2i - 2j + k$ and O is the origin. Find the velocity of the point $3i + 2j - k$ on the body. (8)

b. Forces $2i + 7j$, $2i - 5j + 6k$, $-i + 2j - k$ act at a point P whose position vector is $4i - 3j - 2k$. Find the vector moment of the resultant of three forces acting at P about the point Q , whose position vector is $6i + j - 3k$. (8)

Q.6 a. Solve the differential equation $(D - 1)^2 (D^2 + 1)y = \sin \frac{1}{2}x$, $D = \frac{d}{dx}$ (8)

b. Solve the equation $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 2x$ (8)

Q.7 Examine the following series

(i) $\sum \frac{1}{n^p}$

(ii) $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots$ (16)

Q.8 a. Obtain the Inverse Laplace Transform of $\cot^{-1}\left(\frac{s+3}{2}\right)$ (8)

b. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$ (8)

Q.9 a. Obtain the Laplace transform of $t^2 e^t \sin 4t$. (8)

b. Using the Laplace transforms, find the solution of the initial value problem $y'' + 9y = 6 \cos 3t$, $y(0) = 2$, $y'(0) = 0$ (8)