

DipIETE – ET/CS {NEW SCHEME}

Time: 3 Hours

DECEMBER 2014

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Evaluation of $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$ is equal to

- (A) 0 (B) 1
(C) $\frac{1}{2}$ (D) does not exist

b. If $y = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$, then $\frac{dy}{dx}$ is equal to

- (A) x (B) y
(C) x+y (D) xy

c. The value of integral $\int \frac{dx}{x^2 + 2x + 5}$ is equal to

- (A) $\frac{1}{2} \sin^{-1} \frac{x+1}{2}$ (B) $\frac{1}{2} \cos^{-1} \frac{x+1}{2}$
(C) $\frac{1}{2} \tan^{-1} \frac{x+1}{2}$ (D) $\frac{1}{2} \cot^{-1} \frac{x+1}{2}$

- d. Rank of matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ is
- (A) 4 (B) 1
(C) 3 (D) 2
- e. The differential equation $(2xy + 3y) dx + (x^2 + kx) dy = 0$ is exact if k is equal to
- (A) 3 (B) 4
(C) 2 (D) 6
- f. The middle term in the expansion of $\left(\frac{x}{y} + \frac{y}{x}\right)^6$ is
- (A) 10 (B) 15
(C) 20 (D) 25
- g. The value of $\sin 105^\circ + \cos 105^\circ$ is equal to
- (A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$
(C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{\sqrt{3}}$
- h. The area of the triangle with vertices $(a, 0)$, $(-a, 0)$ and $(0, b)$ is
- (A) a^2 (B) b^2
(C) ab (D) 0
- i. The equation of the straight line which passes through the origin and makes an angle $\frac{3\pi}{4}$ with x – axis is
- (A) $y - 2x = 0$ (B) $y + 2x = 0$
(C) $y - x = 0$ (D) $y + x = 0$
- j. The equation of circle which is concentric with the circle $x^2 + y^2 - 8x + 12y + 43 = 0$ and which passes through $(6, 2)$ is
- (A) $x^2 + y^2 - 8x + 12y - 16 = 0$ (B) $x^2 + y^2 + 8x - 12y + 16 = 0$
(C) $x^2 + y^2 - 8x + 12y + 8 = 0$ (D) $x^2 + y^2 + 8x + 12y - 8 = 0$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. State and prove Leibnitz theorem for the n^{th} derivative of the product of two functions. (8)

b. Find the points at which the function $f(x) = (x-1)(x-2)(x-3)$ has a maximum and minimum values. (8)

Q.3 a. Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ (8)

b. Integrate $\int \frac{x^2 - 3x + 4}{(x-2)(x+2)(x+4)} dx$ (8)

Q.4 a. Compute the inverse of $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ (8)

b. Show that the equations $2x + y + 2z = 1$
 $x + 2y - z = 2$
 $5x + 4y + 3z = 4$
are consistent and solve them. (8)

Q.5 a. Solve the differential equation: (8)

$$\frac{dy}{dx} = \frac{y}{x} - \sqrt{\left(\frac{y^2}{x^2} - 1\right)}$$

b. Solve the differential equation: (8)

$$e^{-y} \sec^2 y dy = dx + x dy$$

Q.6 a. If S_1, S_2, S_3 be the sums of $n, 2n, 3n$ terms respectively of an A.P. show that $S_3 = 3(S_2 - S_1)$ (8)

b. The sum of first and second terms of a G.P. is $\frac{5}{4}$ and the sum of the fourth and fifth terms is 80. Find the first term and the common ratio. (8)

Q.7 a. If $A + B + C = \pi$, show that $\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}$ (8)

b. In any triangle ABC, prove that $\tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$ (8)

Q.8 a. Find the equation of the line through the point of intersection of $5x-3y = 1$ & $2x + 3y = 23$ and perpendicular to the line whose equation is $5x-3y = 1$. (8)

b. If p and p' be the perpendiculars from the origin upon the straight lines whose equations are (8)

$$x \sec \theta + y \operatorname{cosec} \theta = a$$

and $x \cos \theta - y \sin \theta = a \cos 2\theta$

Prove that $4p^2 + (p')^2 = a^2$

Q.9 a. Find the vertex, focus, axis and the directrix of the parabola $y^2 = x + 2y - 2$ (8)

b. Show that the sum of the focal distances of any point on an ellipse is constant and equal to the major axis. (8)