ROLL NO. ________________

Code: AE73 **Subject: INFORMATION THEORY & CODING**

AMIETE – ET

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. For the probability density function of a random variable X given by $f_X(x) = 5e^{-Kx}u(x)$, where $u(x)$ is the unit step function, the value of K is

(A) $\frac{1}{5}$ **(B)** $\frac{1}{25}$
(C) 25 **(D)** 5

b. The spectral density of white noise is

(A) Exponential **(B)** Uniform
(C) Poisson **(D)** Gaussian

c. The auto-correlation function $R_X(\tau)$ of a random process has the property that $R_X(0)$ is equal to

(A) The square of the mean value of the process
(B) The mean squared value of the process
(C) The smallest value of $R_S(\tau)$
(D) ½ $[R_X(\tau) + R_X(-\tau)]$

d. If $f_{XY}(x, y) = f_X(x)\, f_Y(y)$ then X and Y variables are

(A) correlated **(B)** cross-correlated
(C) dependent **(D)** independent

e. In a linear block code, for each $(k\times n)$ generator matrix 'G', there exists a parity check matrix 'H' of size

(A) $(k-n)\times k$ **(B)** $k\times(n-k)$
(C) $(n-k)\times n$ **(D)** $k\times n$

f. The generator matrix for a (7, 4) linear block code is given by:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Then the code word for the message vector [1 0 1 1] will be

(A) 1011001 **(B)** 1000110
(C) 1010101 **(D)** 1010111

g. An optimal decoding method for minimising the probability of sequence error is

(A) the (n-k) stage shift register scheme
(B) the Bahl algorithm
(C) the maximum a posteriori algorithm
(D) the viterbi algorithm

h. A generator polynomial g(X) of an (n,k) cyclic code is a factor of

(A) X^{n+1} **(B)** X^{n-k-1}
(C) $X^{n}+1$ **(D)** $X^{n}-1$

i. The channel capacity of a Gaussian channel of infinite band width is given by (where S is the signal power and $\eta/2$ is the PSD of white noise)

(A) ∞ bits/sec **(B)** $2.88\left(S/\eta\right)$ bits/sec
(C) $1.44\left(S/\eta\right)$ bits/sec **(D)** $\log_2\left(1+S/\eta\right)$ bits/sec

j. The purpose of source coding is to

(A) Increase the information transmission rate
(B) Decrease the information transmission rate
(C) Decrease the S/N ratio
(D) Decrease the probability of error

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that if events **A** and **B** are independent, then
$P\left(A \cap \overline{B}\right) = P(A)P\left(\overline{B}\right)$ **(6)**

b. In a binary communication system (Fig.1), a 0 or 1 is transmitted. Because of channel noise, a 0 can be received as 1 and vice versa. Let m_0 and m_1 denote the events of transmitting 0 and 1, respectively. Let r_0 and r_1 denote the events of receiving 0 and 1, respectively. Let $P(m_0) = 0.5$, $P(r_1|m_0) = p = 0.1$, and $P(r_0|m_1) = q = 0.2$

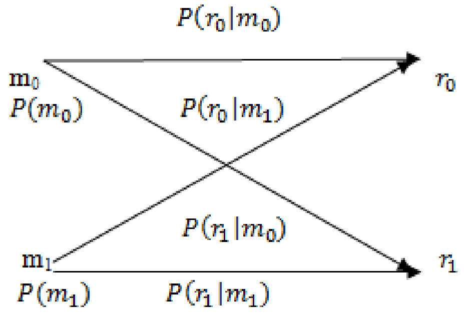

Fig. 1

(10)

(i) Find $P(r_0)$ and $P(r_1)$.
(ii) If a 0 was received, what is the probability that a 0 was sent?
(iii) If a 1 was received, what is the probability that a 1was sent?
(iv) Calculate the probability of error P_e.
(v) Calculate the probability that the transmitted signal is correctly read at the receiver.

Q.3 a. Let X and Y be defined by

$$X = \cos\theta \text{ and } Y = \sin\theta$$

Where θ is a random variable uniformly distributed over $[0,2\pi]$.
(i) Show that X and Y are uncorrelated
(ii) Show that X and Y are not independent **(8)**

b. Consider a random process Y(t) defined by

$$Y(t) = \int_0^t X(\tau)\,d\tau$$

Where X(t) is given by

$$X(t) = A\cos\omega t$$

Where ω is constant and $A = N[0;\sigma^2]$.
(i) Determine the pdf of Y(t) at $t = t_k$
(ii) Is Y(t) WSS? **(8)**

Q4 a. Explain about the average information content of symbols in long dependent sequences. **(8)**

b. Calculate the conditional entropy of an M-ary discrete memoryless channel. **(8)**

Q.5 a. A message source generates one of four messages randomly every microseconds. The probabilities of these messages are 0.4, 0.3, 0.2, and 0.1. Each emitted message is independent of the other messages in the sequence.
(i) What is the source entropy?
(ii) What is the rate of information generated by this source (in bits per second)? **(6)**

b. An analog signal having 4-kHz bandwidth is sampled at 1.25 times the Nyquist rate, and each sample is quantized into one of 256 equally likely levels. Assume that the successive samples are statistically independent.
(i) What is the information rate of this source?
(ii) Can the output of this source be transmitted without error over an AWGN channel with a bandwidth of 10 kHz and S/N ratio of 20 dB?
(iii) Find the S/N ratio required for error free transmission for part (ii) **(10)**

Q.6 a. Determine the capacity of a channel of infinite bandwidth. **(8)**

b. Consider a binary memoryless source X with two symbols x_1 and x_2. Show that H(X) is maximum when both x_1 and x_2 are equiprobable. **(8)**

Q.7 a. Consider an AWGN channel with 4-kHz bandwidth and the noise power spectral density $\frac{\eta}{2} = 10^{-12}$ W/Hz. The signal power required at the receiver is 0.1 mW. Calculate the capacity of this channel. **(8)**

b. Draw and explain observations of Bandwidth-Efficiency diagram. **(8)**

Q.8 a. For a (6, 3) systematic linear block code, the three parity check bits c_4, c_5, and c_6 are formed from the following equation:

$$c_4 = d_1 \oplus d_3$$
$$c_5 = d_1 \oplus d_2 \oplus d_3$$
$$c_6 = d_1 \oplus d_2$$

(i) Write down the generator matrix G.
(ii) Construct all possible code words.
(iii) Suppose that the received word is 010111. Decode this received word by finding the location of the error and the transmitted data bits. **(10)**

b. Given a generator matrix G = [1 1 1 1 1]. Construct a (5, 1) code. How many errors can this code correct? Find the codeword for data vectors d=0 and d=1. Comment on the result. **(6)**

Q.9 a. Draw the state diagram, tree diagram, and trellis diagram for the K=3, rate 1/3 code generated by

$$g_1(X) = X + X^2$$
$$g_2(X) = 1 + X$$
$$g_3(X) = 1 + X + X^2$$ **(12)**

b. Factorize the polynomial $x^3 + x^2 + x + 1$ **(4)**