Code: AE57/AC57/AT57 Subject: SIGNALS AND SYSTEMS

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The Fourier transform of the exponential signal $e^{-j\omega_0 t}$ is:

(A) a constant **(B)** a rectangular pulse
(C) an impulse **(D)** impulse train

b. The system characterized by the equation y(t) = a x(t)+b is

(A) linear if b < 0 **(B)** linear if b > 0
(C) linear for any value of b **(D)** non- linear

c. A band pass signal extends from 1 KHz to 4 KHz. The minimum sampling frequency needed to retain all information in the sampled signal is

(A) 1 KHz **(B)** 6 KHz
(C) 3 KHz **(D)** 4 KHz

d. The Fourier Transform of a rectangular pulse existing between t = –T/2 to t = T /2 is a

(A) $(\text{sinc})^2$ function **(B)** sinc function
(C) $(\text{sine})^2$ function **(D)** sine function

e. δ(n–N) * δ(n+ N) will result in

(A) 0 **(B)** N
(C) Always 1 **(D)** can't decide from given data

f. Fourier Transform pair x(t) ↔ 2π(–w) represents ______ property.

(A) Duality **(B)** Time-reversal
(C) Symmetry **(D)** Both **(A)** and **(B)**

g. The discrete-time signal x(n) shown in Fig.1 is periodic with fundamental period

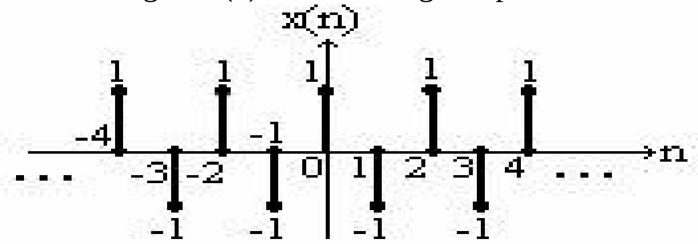

Fig.1

(A) 6 **(B)** 4
(C) 2 **(D)** 0

h. The impulse response of a system is $h(n) = a^n\, u(n)$. The condition for the system to be stable is

(A) a is real and positive **(B)** a is real and negative
(C) $|a| < 1$ **(D)** $|a| > 1$

i. The ROC does not contain any

(A) other ROC **(B)** unit circle
(C) zeros **(D)** poles

j. For a Random Variable f(x), the integral $\int x\, f(x)\, dx$, defines

(A) variance **(B)** mean
(C) pdf **(D)** co-variance

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $x(t) = u(t) - {}^1/_2$; **(4)**

(i) Sketch x(t)

(ii) Determine analytically the signal is periodic or not if periodic, state the period

b. Do as directed: **(3×3)**

(i) Plot x(n) and also plot y[n] = x[n+1] where x[n] is defined as below:

$$x[n] = \begin{cases} 0 & \text{if} \quad n < 2 \\ 2n-4 & \text{if} \quad 2 \le n \le 4 \\ 4-n & \text{if} \quad 4 \le n \end{cases}$$

(ii) Determine convolution of $x(t) = e^{2t}u(-t)$ and $h(t) = u(t-3)$ and plot resultant y(t).

(iii) Let h(n) be the impulse response of the LTI causal system described by the difference equation
$y(n) = a\, y(n-1) + x(n)$ and let $h(n) * h_1(n) = \delta(n)$.
Find $h_1(n)$.

c. Differentiate between Energy and Power signal. **(3)**

Q.3 a. Compute the Fourier series for the following signal shown in Fig.2. **(8)**

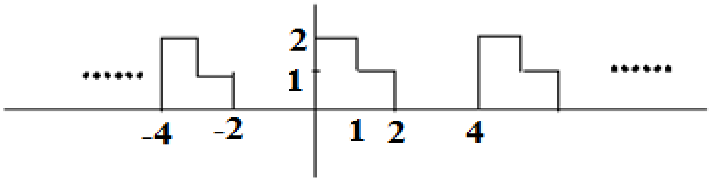

Fig.2

b. Consider an LTI system with impulse response $h(n) = \alpha^n u(n)$, $-1 < \alpha < 1$; and with the input $x(n) = \cos(2\pi n/N)$. Determine $y(n)$. **(8)**

Q.4 a. State and explain convergence conditions for continuous–time Fourier transform. **(4)**

b. Consider a stable LTI system characterized by the differential equation: $dy(t)/dt + 5\,y(t) = x(t)$. Determine the (i) frequency response and (ii) impulse response for the system. **(4×2)**

c. State and prove following properties for continuous time Fourier transforms:
(i) Conjugation
(ii) Time shifting
(iii) Differentiation
(iv) Duality **(4)**

Q.5 a. Consider a stable Causal LTI system whose input $x(n)$ and output $y(n)$ are related through second order difference equation
$y(n) - (3/4)\,y(n-1) + (1/8)\,y(n-2) = 2\,x(n)$; determine the response for the given input $x(n) = (1/4)^n u(n)$ **(8)**

b. For signal $x(n) = \cos w_o n$ with $w_o = 2\pi/5$, obtain and plot $X(e^{jw})$. **(4)**

c. State and prove following properties for discrete time Fourier transforms:
(i) Time shifting
(ii) Frequency shifting **(2×2)**

Q.6 a. Explain the concept of : **(4×2)**
(i) Non-linear phase
(ii) Group delay
(iii) Continuous-time ideal low pass filter
(iv) First order continuous time system

b. Define sampling, aliasing and Nyquist interval. For the following signal x (t), calculate Nyquist rate.
$x(t) = 6\cos 50\pi t + 20\sin 300\pi t - 10\cos 100\pi t$ **(8)**

Q.7 a. Discuss the all-pass system using Laplace transform with necessary diagrams. **(6)**

b. Obtain the Laplace transform of:
(i) $x(t) = e^{-at} u(t)$ (ii) $x(t) = -e^{-at} u(-t)$ **(2×3)**

c. State initial value and final value theorems for Laplace Transform. Also state its usefulness. **(4)**

Q.8 a. Let the z- transform of x(n) be X(z). Show that the z-transform of x (-n) is X(1/z) **(4)**

b. Determine the region of convergence of the z-transform of the signal
$x(n) = 2^n u(n) - 3^n u(-n-1)$ **(4)**

c. Find Inverse Z-Transform of following: **(2×4)**
(i) $X(z) = 1/(1 - az^{-1})$, $|z| > |a|$
(ii) $X(z) = \log(1 + az^{-1})$, $|z| > |a|$

Q.9 a. Explain the following random processes: **(4×2)**
(i) ergodic
(ii) non-ergodic
(iii) stationary
(iv) non-stationary

b. A random variable V = b + x ; where x is a Gaussian distributed random variable with mean 0 and variance σ^2 with ‘ b’ a constant. Show that V is a Gaussian distributed random variable with mean b and variance σ^2. **(8)**