ROLL NO. ______________

Code: AE56/AC56/AT56 Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $\omega = \log z$, then $\frac{d\omega}{dz}$ is equal to

(A) $\frac{1}{z}$ **(B)** $-\frac{1}{z}$

(C) z **(D)** $-z$

b. The value of the integral $\int_C \frac{z^2 - z + 1}{z - 1} dz$, where C is the circle and $|z| = 2$

(A) πi **(B)** $2\pi i$

(C) $3\pi i$ **(D)** $4\pi i$

c. The directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $P(1,1,1)$ in direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$, is

(A) $\frac{11}{3}$ **(B)** $\frac{11}{2}$

(C) $11\frac{2}{3}$ **(D)** $\frac{2}{3}$

d. The value of $\int_C \vec{f}\,\vec{dr}$ where $\vec{f} = 2x^2y\hat{i} + 3xy\hat{j}$ and C is $y = 4x^2$ in the plane from $(0,0)$ to $(1,4)$ is

(A) $\frac{104}{9}$ **(B)** $\frac{104}{7}$

(C) $\frac{104}{3}$ **(D)** $\frac{104}{5}$

e. The relation between E and Δ is

(A) $\Delta = 1 + E$ **(B)** $E = 1 - \Delta$
(C) $E = 1 + \Delta$ **(D)** $\Delta = -1 - E$

f. The solution of $(E^2 + 6E + 9)y_n = 0$ is

(A) $y_n = (c_1 + c_2 n)(-3)^n$ **(B)** $y_n = (c_1 - c_2 n)(3)^n$
(C) $y_n = (c_1 + c_2)(-3)^n$ **(D)** $y_n = (c_1 \cdot c_2)(3)^n$

g. The solution of Partial Differential Equation of $p^2 + q^2 = 1$ is

(A) $z = ax - \sqrt{1-a^2} \cdot y + c$ **(B)** $z = ax + \sqrt{1-a^2} \cdot y + c$
(C) $z = ax + \sqrt{1+a^2} \cdot y + c$ **(D)** $z = ax - \sqrt{1+a^2} \cdot y + c$

h. From a pack of 52 cards, one is drawn at random, then the probability of getting a king is

(A) $\frac{1}{11}$ **(B)** $\frac{1}{12}$
(C) $\frac{1}{13}$ **(D)** $\frac{1}{14}$

i. The probability of getting 4 heads in 6 tosses of a fair coin is

(A) $\frac{15}{64}$ **(B)** $\frac{17}{64}$
(C) $\frac{19}{64}$ **(D)** $\frac{21}{64}$

j. The probability that the ace of spades will be drawn from a pack of well shuffled cards at least once in 104 consecutive trials is, (using Poisson distribution)

(A) 0.468 **(B)** 0.648
(C) 0.0864 **(D)** 0.864

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof. **(8)**

b. Find the bilinear transformation which maps the points $z_1 = 2, z_2 = i$ and $z_3 = -2$ into the points $w_1 = 1, w_2 = i$ and $w_3 = -1$. **(8)**

Q.3 a. Find the Taylor's series expansion of a function of the complex variable $f(z)=\dfrac{1}{(z-1)(z-3)}$ about the point, z = 4. Find its region of convergence. **(8)**

b. Determine the poles of the function $f(z)=\dfrac{z^2}{(z-1)^2(z+2)}$ and find residue of $f(z)$ of at each pole. **(8)**

Q.4 a. Evaluate grad e^{r^2} where $r^2=x^2+y^2+z^2$. **(8)**

b. Prove that $div\left\{\dfrac{f(r)\cdot r}{r}\right\}=\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 f\right)$ **(8)**

Q.5 a. Apply Green's theorem to evaluate $\oint_C 2y^2dx+3xdy$

Where C is the boundary of closed region bounded between y = x and $y=x^2$ **(8)**

b. Apply Stoke's theorem to calculate $\oint_C 4ydx+2zdy+6ydz$

where C is the curve of intersection of $x^2+y^2+z^2=6z$ and $z=x+3$ **(8)**

Q.6 a. Using Langrange's interpolation formula, find the value of y corresponding to x = 10 from the following table: **(8)**

x	5	6	9	11
y	12	13	14	16

b. Approximate the integral, $I=\int_0^1 e^x dx$ using Simpson's $^1/_3$rd Rule with n = 8 interval. Round off the result at 4 digits. **(8)**

Q.7 a. Apply Charpit method to solve the equation $px+qy=pq$ **(8)**

b. Using the method of separation of variable, solve $\dfrac{\partial u}{\partial x}=2\dfrac{\partial u}{\partial t}+u$, where $u(x,0)=6e^{-3x}$ **(8)**

Q.8 a. An urn 'A' contains 2 white and 4 black balls. Another urn 'B' contains 5 white and 7 black balls. A ball is transferred from the urn 'A' to the urn 'B', then a ball is drawn from urn 'B'. Find the probability that it is white. **(8)**

b. Assuming half the population of a town consumes chocolates and that 100 investigators each take 10 individuals to see whether they are consumers, how many investigators would you expect to report that three people or less were consumers? **(8)**

Q.9 a. If the variance of the Poisson distribution is 2, find the probabilities for $r = 1,2,3,4$ from the recurrence relation of the Poisson distribution. Also find $P(r \geq 4)$. **(8)**

b. The life of army shoes is 'normally' distributed with mean 8 months and standard deviation 2 months. If 5000 pairs are issued how many pairs would be expected to need replacement after 12 months? [Given that $\mathrm{P}(z \geq 2) = 0.0228$ and $z = \dfrac{x - \mu}{\sigma}$] **(8)**