ROLL NO. ___________________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS - I

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $z = \sin^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then the value of $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}$ is equal to

(A) sin z **(B)** cos z
(C) tan z **(D)** cot z

b. The value of integral $\iint xy \; dx\, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ is

(A) a^4 **(B)** $\frac{a^4}{2}$

(C) $\frac{a^4}{4}$ **(D)** $\frac{a^4}{8}$

c. If the product of two eigen values of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16, then the third eigen value is

(A) 2 **(B)** –2
(C) 3 **(D)** –3

d. Taylor's series solution of $\frac{dy}{dx} = -xy, y(0) = 1,$ upto x^2 is

(A) $y(x) = 1 + x + \frac{x^2}{2} + ...$ **(B)** $y(x) = 1 + x - \frac{x^2}{2} + ...$

(C) $y(x) = 1 + \frac{x^2}{2} + ...$ **(D)** $y(x) = 1 - \frac{x^2}{2} + ...$

e. Integrating factor of the differential equation $(1+x)\dfrac{dy}{dx} = 2y + (x+1)^4$ is

(A) $(1 + x)^2$ **(B)** $\dfrac{1}{(1+x)^2}$

(C) $(1+ x)$ **(D)** $\dfrac{1}{1+x}$

f. The solution of $4\dfrac{d^3x}{dt^3} + 4\dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} = 0$, is

(A) $x(t) = (C_1 + C_2 t)^{t/2} + C_3$ **(B)** $x(t) = C_1 + C_2 t + C_3 e^{t/2}$
(C) $x(t) = (C_1 + C_2 t)e^{-t/2} + C_3$ **(D)** $x(t) = C_1 + C_2 t + C_3 e^{-t/2}$

g. Particular integral of the differential equation $(D - 2)^2 = e^{2x}$, is

(A) $\dfrac{x^2}{2}e^{2x}$ **(B)** $\dfrac{x^2}{4}e^{2x}$

(C) xe^{2x} **(D)** $\dfrac{x}{2}e^{2x}$

h. The value of $\beta\left(\dfrac{5}{2}, \dfrac{5}{2}\right)$ is

(A) $\dfrac{3\pi}{192}$ **(B)** $\dfrac{3\pi}{128}$

(C) $\dfrac{3\pi}{64}$ **(D)** $\dfrac{3\pi}{50}$

i. $J_{½}(x)$ is equal to

(A) $J_{-\frac{1}{2}}(x)\cos x$ **(B)** $J_{-\frac{1}{2}}(x)\sin x$

(C) $J_{-\frac{1}{2}}(x)\cot x$ **(D)** $J_{-\frac{1}{2}}(x)\tan x$

j. The polynomial $3x^2+2x$ in terms of Legendre polynomials is

(A) $3P_2(x) + 2P_1(x)$ **(B)** $2 P_2(x) + 2P_1(x)$
(C) $2 P_2 (x) + 2 P_1 (x) + P_0 (x)$ **(D)** $2 P_2 (x) + P_1 (x) + P_0 (x)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $r^2 = x^2 + y^2 + z^2$ and $V = r^m$, prove that
$V_{xx} + V_{yy} + V_{zz} = m(m+1)\, r^{m-2}$ **(8)**

b. If xyz = 8, find the values of x, y, z for which $u = \dfrac{5xyz}{x+2y+4z}$ is a maximum. **(8)**

Q.3 a. Evaluate by changing order of integration of $\int_0^3 \int_1^{\sqrt{4-y}} (x+y)\,dx\,dy$ **(8)**

b. Find the volume of the tetrahedron bounded by the coordinate planes and the plane $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$ **(8)**

Q.4 a. Show that if $\lambda \neq -5$, the system of equations,
$3x - y + 4z = 3,$
$x + 2y - 3z = -2,$
$6x + 5y + \lambda z = -3,$
have a unique solution. If $\lambda = -5$, then show that the equations are consistent. Find the solutions in each case. **(8)**

b. Use Gauss elimination method to solve the equations,
$$x - y + z = 6$$
$$2x + 4y + z = 3$$
$$3x + 2y - 2z = -2$$ **(8)**

Q.5 a. Develop Newton – Raphson formula for finding $\sqrt{N}$, where N is a real number. Use it to find $\sqrt{41}$. Correct to 3 decimal places. **(8)**

b. Use Runge – Kutta method of order four for find y at x = 0.2 given that $\dfrac{dy}{dx}=\dfrac{y^2-x^2}{y^2+x^2}, y(0)=1$
taking h = 0.2 **(8)**

Q.6 a. Solve the differential equation $(1+x+y)^2\dfrac{dy}{dx}=1$ **(8)**

b. Show that the family of parabolas $x^2 = ua\,(y + a)$ is self orthogonal. **(8)**

Q.7 a. Solve the equation $(D^3 + 2D^2 + D)y = x^2 e^{2x} + \sin^2 x$ **(8)**

b. Use method of variation of parameters to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$ **(8)**

Q.8 a. Find the series solution of the differential equation

$$2x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + (1-x^2)y = 0$$ **(8)**

b. Show that $\int_a^b (x-a)^m (b-x)^n dx = (b-a)^{m+n+1}\beta(m+1, n+1)$ **(8)**

Q.9 a. Prove that $P_n(x) = \frac{1}{\lfloor n} \frac{1}{2^n} \frac{d^n}{dx^n}\left(x^2 - 1\right)^n$ **(8)**

b. Show that :

$$\frac{d}{dx}\left\{J_n^2(x) + J_{n+1}{}^2(x)\right\} = 2\left\{\frac{n}{x}J_n^2(x) - \frac{n+1}{x}J_{n+1}^2(x)\right\}$$ **(8)**