AMIETE – ET/CS/IT *{NEW SCHEME}*

Time: 3 Hours   **DECEMBER 2014**   **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1   **Choose the correct or the best alternative in the following:**   **(2×10)**

a. The value of $\int_c \log z\, dz$, where c is the unit circle $|z| = 1$

   (A) $2\pi i$ **(B)** $-2\pi i$
   (C) 1 **(D)** -1

b. The residue at simple pole of the function $f(z) = \dfrac{z^2}{(z-1)^2(z+2)}$ is

   (A) $\dfrac{1}{2}$ **(B)** $\dfrac{-1}{2}$

   (C) $\dfrac{4}{9}$ **(D)** 1

c. If $\vec{F}$ is such that $\nabla \times \vec{F} = 0$ then $\vec{F}$ is called

   (A) Rotational   **(B)** Irrotational
   (C) Solenoidal   **(D)** None of these

d. If $\Phi(x, y, z) = 3xyz^2 - y^3 z^2$, then the value of grad Φ at point $(1,-2,-1)$ is

   (A) 0   **(B)** 2
   (C) –2   **(D)** –16i –9j – 4k

e. If $\vec{A} = x\,i - x^2 j + (x-1)\,k$ and $\vec{B} = 2\,x^2 i + 6xk$, then the value of $\int_0^2 \vec{A} \bullet \vec{B}\, dx$

   is

   (A) 5   **(B)** 12
   (C) 7   **(D)** –5

f. The value of $\int_{0}^{6} \dfrac{dx}{1+x^2}$ by trapezoidal rule is

(A) 1   (B) –1
(C) 1.4108   (D) 5

g. The solution of $xp + yq = z$ is

(A) $f(\dfrac{x}{y}, \dfrac{y}{z}) = 0$ (B) $f(x, y) = 0$

(C) $f(xy, yz) = 0$ (D) $f(x^2 + y^2) = 0$

h. A five-digit number is formed by using the digit 0,1,2,3 and 5 without repetition, the probability of the number that is divisible by 6 is

(A) 1   (B) –1

(C) $\dfrac{25}{4}$ (D) None of these

i. If three persons selected at random are stopped on a street, then the probability that all of them were born on Sunday is

(A) $\dfrac{1}{35}$ (B) $\dfrac{1}{343}$

(B) $\dfrac{3}{352}$ (D) None of these

j. In the Poisson distribution, if $2P\,(x = 1) = P(x = 2)$, then mean is

(A) 4   (B) 1
(C) –1   (D) None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2   a.   Show that the real and imaginary parts of the function $w = \log z$ satisfy the Cauchy – Riemann equations when z is not zero. Find its derivatives.   **(8)**

   b.   Evaluate $\int_{0}^{1+i}(x^2 - iy)\,dz$ along the path (i). y = x   (ii). y = x². **(8)**

Q.3   a.   Find the bilinear transformation, which maps $z_1 = 0$, $z_2 = 1$, $z_3 = \infty$ in to **(8)**
$w_1 = i, w_2 = -1, w_3 = -i$

   b.   Find the terms in the Laurent expansion of $f(z) = \dfrac{1}{(z+1)(z+3)}$, for the region

   (i) $1 < |z| > 3$ (ii) $|z| < 1$ **(8)**

Q.4   a.   If $\vec{r} = xi + yj + zk$, $a = \left|\vec{r}\right|$ and $\vec{a}$ is a constant vector, then find the value of

$$div\left[\frac{\vec{a} \times \vec{r}}{r^n}\right]$$ **(8)**

b   Define CURL of a vector point function with physical interpretation.   **(8)**

Q.5   a.   Evaluate $\int_c \vec{F} \bullet d\vec{r}$, where $\vec{F} = \dfrac{iy - jx}{x^2 + y^2}$ and c is the circle $x^2 + y^2 = 1$ traversed counter clockwise.   **(8)**

b.   Verify Stocke's Theorem for $\vec{F} = (x^2 + y - 4)i + 3xyj + (2xz + z^2)k$ over the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above the x-y plane.   **(8)**

Q.6   a.   State and Prove Lagrange's interpolation formula.   **(8)**

b.   Evaluate $\displaystyle\int_{0.5}^{0.7} \sqrt{x}\, e^{-x}\, dx$ by Simpson's $\dfrac{1}{3}$ rule.   **(8)**

Q.7   a.   Form the partial differential equation by eliminating the function f from the relation $z = y^2 + 2\,f(\dfrac{1}{x} + \log y)$.   **(8)**

b.   Solve $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial x \partial y} - 6\dfrac{\partial^2 z}{\partial y^2} = y\cos x$ **(8)**

Q.8   a.   State and prove BAYE'S theorem.   **(8)**

b.   A can hit a target 4 times in 5 shots, B 3 times in 4 shots, C twice in 3 shots. They fire a volley. What is the probability that at least two shots hit?   **(8)**

Q.9   a.   Out of 800 families with 4 children each, how many families would be expected to have   (i) 2 boys and 2 girls   (ii) At least one boy
                           (iii) No girl   (iv) At most two girls?   **(8)**
Assume equal probability for boys and girls

b.   If there are 3 misprints in a book of 1000 pages find the probability that a given page will contain
(i) no misprint   (ii) more than 2 misprints   **(8)**