ROLL NO. ___________________

Code: AE101/AC101/AT101 **Subject: Engineering Mathematics-I**

AMIETE – ET/CS/IT *{NEW SCHEME}*

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $x+y+z=\log z$, then the value of $\frac{\partial z}{\partial x}$ is

(A) $\frac{x}{1-x}$ **(B)** $\frac{z}{1-z}$

(C) $\frac{y}{1-y}$ **(D)** None of these

b. The Jacobian $J\left(\frac{x,y}{r,\theta}\right)$ for the function $x=r\cos\theta,\ y=r\sin\theta$ is

(A) r **(B)** 1

(C) $\frac{1}{r}$ **(D)** None of these

c. Matrix has the value. This statement

(A) is always true **(B)** dependent upon the matrix

(C) is false **(D)** None of these

d. The value of $\int_0^\infty e^{-x^2}dx$ is

(A) π **(B)** $-\pi$

(C) $\frac{\sqrt{\pi}}{2}$ **(D)** None of these

e. If the Fourier series of f (x) has only sine terms, then f (x) must be

(A) Odd **(B)** Even
(C) Both **(A)** and **(B)** **(D)** None of these

f. The value of $J_{-\frac{1}{2}}(x)$ is

(A) $\sqrt{\frac{2}{\pi x}}\cos x$ **(B)** $\sqrt{\frac{2}{\pi x}}\sin x$

(C) $\sqrt{\frac{2}{\pi}}\sin x$ **(D)** None of these

g. The value of $\int_{-1}^{+1} P_m(x)\,P_n(x)\,dx$ for $m \neq n$ is

(A) –1 **(B)** 0
(C) 1 **(D)** None of these

h. The order of convergent in Newton-Rapson method is

(A) 0 **(B)** 3
(C) 2 **(D)** None of these

i. The solution of the differential equation $(D^2 + 6D + 9)y = 5e^{3x}$ is

(A) $y = (c_1 + xc_2)\,e^{-3x} + \frac{5\,e^{3x}}{36}$ **(B)** $y = (c_1 - xc_2)\,e^{3x} - \frac{5}{36}$

(C) 0 **(D)** None of these

j. The value of $\int_0^1\int_0^x (x^2 + y^2)dx\,dy$ is

(A) 5 **(B)** $\frac{1}{3}$

(C) $\frac{7}{2}$ **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$ **(8)**

b. Evaluate $\int_0^1 \frac{x^{\alpha} - 1}{\log x}dx,\ \alpha \geq 0,$ by using the method of differention under the sign of integration. **(8)**

Q.3 a. Expand $f(x) = x^3$ *as* a Fourier series in the interval $-\pi < x < \pi$. **(8)**

b. Obtain the half range cosine series for $\sin\left(\frac{\pi x}{l}\right)$ in the range $0 < x < l$. **(8)**

Q.4 a. Find the Fourier sine transform of $\frac{1}{x(x^2 + a^2)}$. **(8)**

b. Find the Z-transform of sin (3k+5) **(8)**

Q.5 a. Express $f(x) = 4x^3 - 2x^2 - 3x + 8$ in term of Lagrange polynomials. **(8)**

b. Prove that $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ **(8)**

Q.6 a. Find the real root of $x^3 - 2x - 5 = 0$, correct to three decimal places using Newton-Rapson Method. **(8)**

b. Apply R.K Method of fourth order, to find an approximate value of y when x = 0.1. Given that $10\frac{dy}{dx} = x^2 + y^2, y(0) = 1.$ **(8)**

Q.7 a. Solve the differential equation $x^2 \frac{d^2 y}{dx^2} - 3x\frac{dy}{dx} + 5y = x \log x$. **(8)**

b. Solve $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = x\, e^x \sin x$ **(8)**

Q.8 a. Find the rank of the matrix A, where A= $\begin{bmatrix} 1 & 3 & 4 & 2 \\ 2 & -1 & 3 & 2 \\ 3 & -5 & 2 & 2 \\ 6 & -3 & 8 & 6 \end{bmatrix}$, by reducing it to normal form. **(8)**

b. Investigate the value of λ *and* μ so that the equations $2x + 3y + 5z = 9;\ 7x + 3y - 2z = 8;\ 2x + 3y + \lambda z = \mu$ **(8)**
(i) No solution (ii) Unique solution (iii) An infinite number of solutions

Q.9 a. Evaluate $\int_0^\infty \int_0^\infty x\, e^{-\frac{x^2}{y}}\, dx dy$ **(8)**

b. Define Beta and Gamma functions. Prove that $\beta(m,n) = \frac{\gamma(m)\gamma(n)}{\gamma(m+n)}$. **(8)**