ROLL NO. ___________________

Code: AC65 **Subject: DISCRETE STRUCTURES**

AMIETE – CS

Time: 3 Hours **DECEMBER 2014** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If S and T are two sets, then $|(S \cup T)|$ is:

(A) $|S| + |T|$ **(B)** $|S| + |T| - |S \cap T|$
(C) $|(S \cup T)| - |S \cap T|$ **(D)** None of these

b. The total number of words formed with n distinct letters are:

(A) n **(B)** n.(n – 1)
(C) n.(n – 1).(n – 2)…3.2.1 **(D)** n.(n – 1)/2

c. Given A = {{a}, a, {a, b}}. Which of the following is true:

(A) $a \subseteq A$ **(B)** $b \in A$
(C) $\{a,b\} \subseteq A$ **(D)** $\{a,b\} \in A$

d. Let Q be the set of rational numbers and define a*b = a + b – ab. Structure <Q, *> is:

(A) Semigroup **(B)** Group
(C) Monoid **(D)** None of these

e. In how many ways can 3 boys and 2 girls sit in a row:

(A) 48 **(B)** 120
(C) 6 **(D)** 24

f. The inverse of $p \rightarrow (p \vee \sim q)$ is:

(A) $\sim p \rightarrow (\sim p \wedge q)$ **(B)** $p \wedge (p \wedge \sim q)$
(C) $\sim (p \vee \sim q) \rightarrow p$ **(D)** $p \vee (p \leftrightarrow q)$

g. Let f : R→R be defined by $f(x) = x^2 + 1$, then $f^{-1}(5)$ is:

(A) {2, 3} **(B)** {-2, 2}
(C) {3, 3} **(D)** {-3, 3}

h. Let I^+ be the set of positive integers and R be the relation on I^+ defined by xRy iff $2x \leq y-1$. Then which ordered pair belongs to R:

(A) (2, 2) **(B)** (3, 9)
(C) (3, 2) **(D)** (9, 3)

i. The set of all positive rational numbers forms an abelian group under the composition defined by a * b = ab / 2. Identity of this group is:

(A) 1 **(B)** 0
(C) 2 **(D)** None of these

j. $(P \rightarrow Q) \wedge P$ is logically equivalent to:

(A) P **(B)** Q
(C) $P \rightarrow Q$ **(D)** $P \wedge Q$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Let A = {Φ, b}, construct the following sets: **(8)**

(i) A – Φ
(ii) {Φ} – A
(iii) $A \cap P(A)$
(iv) Φ - A

b. Prove that (i) $(A \cap B) \cup (A \cap {\sim}B) = A$ **(8)**
(ii) $A \cap ({\sim}A \cup B) = A \cap B$

Q.3 a. (i) Given the value of $p \rightarrow q$ is true. Determine the value of ${\sim} p \vee (p \leftrightarrow q)$. **(8)**
(ii) Is the statement tautology?
$(p \wedge (p \rightarrow q)) \rightarrow q$

b. Define logical equivalence. Construct the truth tables for the propositions **(8)**
$(P \vee Q) \wedge {\sim} (P \wedge Q)$ and $(P \wedge {\sim}Q) \vee ({\sim}P \vee Q)$.
Also deduce that whether the above pairs are equivalent.

Q.4 a. Define quantifiers. Negate the statement: **(8)**
For all real x, if x > 3, then $x^2 > 9$.

b. Define the validity of the following argument: **(8)**
"If Ram runs for office, he will be selected. If Ram attends the meeting, he will run for office. Either Ram will attend the meeting or he will go to London. But Ram cannot go to London. Thus Ram will be selected."

Q.5 a. Define Cartesian product on sets. For the given sets X = {1, 2}, Y = {a, b, c} and Z = {c, d}, find $(X \times Y) \cap (X \times Z)$. **(8)**

b. Let a and b be positive integers, and suppose A is defined recursively as follows:

$$A(a,b)=\begin{cases}0 & ,if\ a<b\\ A(a-b,b)+1, & if\ b\le a\end{cases}$$

(i) Find: (i) A (2, 5), (ii) A(12, 5).
(ii) What does this function A do? Find A (5861, 7). **(8)**

Q.6 a. Let R be binary relation on the set of all strings of 0s and 1s such that R = {(a, b) | a and b are strings that have same number of 0s}:

(i) Is R reflexive?
(ii) Is R symmetric?
(iii) Is R transitive?
(iv) Is R a partial order relation? **(8)**

b. Prove that if a and b are elements in a bounded distributive lattice and if a has a compliment a', then
(i) $a\vee(a'\wedge b)=a\vee b$
(ii) $a\wedge(a'\vee b)=a\wedge b$ **(8)**

Q.7 a. Let f : R → R be a function defined as, f(x) = |x|. Show that f is neither one-one nor onto function. **(8)**

b. Define composite functions. Let f : R → R be a function given as, $f(x) = 2-x^2$ and g : $R^+ \to R^+$ be given as $g(x)=\sqrt{x}$, where R^+ is the set of non-negative real numbers. Compute fog(x) and gof(x). **(8)**

Q.8 a. If Z_n denotes the set of integers {0, 1, 2, …., n-1} and * be binary operation on Z_n such that a*b = the remainder of ab divided by n,
(i) Construct the table for the operation * for n = 4
(ii) Show that $(Z_n, *)$ is a semi-group for n = 4. **(8)**

b. Let (R, +) be the additive group of real numbers and $(R^+, \times)$ be a multiplicative group of positive real numbers. Prove that f : R → R^+, defined by $f(x) = e^x$, for all x in R is an isomorphism from (R, +) to $(R^+, \times)$. **(8)**

Q.9 a. The generating function of an encoding function $E: Z_2^3 \to Z_2^6$ is given by

$$G=\begin{bmatrix}1&0&0&1&1&0\\0&1&0&0&1&1\\0&0&1&1&0&1\end{bmatrix}$$

(i) Find the code words assigned to 110 and 010.
(ii)Obtain associated parity-check matrix.
(iii)Hence decode the received word 110110. **(8)**

b. Let n be an integer satisfying n > 1. Then prove that the ring Z_n of congruence classes of integer modulo n is an integral domain if and only if n is a prime number. **(8)**