

DiplETE - ET/CS

Time: 3 Hours

DECEMBER 2013

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of the limit $\lim_{x \rightarrow 0} \frac{\sin x^\circ}{x}$ is equal to

- (A) π (B) 1
(C) $\frac{\pi}{180}$ (D) 0

b. The value of definite integral $\int_0^{\pi/2} \sin^7 x dx$ is equal to

- (A) $\frac{14}{35}$ (B) $\frac{16}{35}$
(C) $\frac{17}{35}$ (D) $\frac{11}{35}$

c. The complementary function for the differential equation $\frac{d^2x}{dt^2} + \frac{g}{\ell}x = \frac{g}{\ell}L$ where g, ℓ, L are constants, is given by

- (A) $c_1 \cos \sqrt{\frac{\ell}{g}} t + c_2 \sin \sqrt{\frac{\ell}{g}} t$ (B) $c_1 \cos \sqrt{\frac{g}{\ell}} t + c_2 \sin \sqrt{\frac{g}{\ell}} t$
(C) $c_1 \cos \sqrt{\frac{g}{\ell}} t + c_2 \sin \sqrt{\frac{g}{\ell}} t$ (D) $c_1 \cos \sqrt{\frac{g}{\ell}} t + ic_2 \sin \sqrt{\frac{g}{\ell}} t$

d. The principal argument of $-2i$ is equal to

- (A) $-\pi/3$ (B) $-\pi/2$
(C) $\pi/2$ (D) $\pi/3$

b. If f is a real function defined by $f(x) = \frac{x-1}{x+1}$ then prove that

$$f(2x) = \frac{3f(x)+1}{f(x)+3} \quad (8)$$

Q.3 a. Find the volume of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle. (8)

b. Find the length of the curve $y^2 = x^3$ from origin to the point (1, 1). (8)

Q.4 a. If n is a positive integer then show that $(\sqrt{3} + i)^n + (\sqrt{3} - i)^n = 2^{n+1} \cos \frac{n\pi}{6}$
where $i = \sqrt{-1}$ (8)

b. A resistance of 20 ohms and inductance of 0.2 H and a capacitance of $100 \mu\text{F}$ are connected in series across 220 Volt, 50 cycle/sec main. Determine:
(i) impedance (ii) current
(iii) voltage across L, R and C (iv) power in watt
(v) power factor (8)

Q.5 a. A rigid body is spinning with an angular velocity of 27 radian/second about an axis parallel to $2i + j - 2k$ passing through the point $i + 3j - k$. Find the velocity of the point whose position vector is $4i + 8j + k$. (8)

b. Find the area of the triangle formed by the point whose position vectors are $3i + j$, $5i + 2j + k$, $i - 2j + 3k$. (8)

Q.6 a. Solve $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 6e^{3x} + 7e^{-2x} - \log 2$ (8)

b. Solve $\frac{d^2y}{dx^2} + 9y = \sec 3x$ (8)

Q.7 a. Expand $f(x) = e^x$ in a cosine series over (0, 1) (8)

b. Find the Fourier Series of the function (8)

$$f(t) = \begin{cases} 0 & \text{when } -2 < t < -1 \\ K & \text{" } -1 < t < 1 \\ 0 & \text{" } 1 < t < 2 \end{cases}$$

Q.8 a. Evaluate $L \left\{ te^{-t} \cosh t \right\}$ (8)

b. Evaluate $L \left\{ \int_0^t \frac{e^t \sin t}{t} dt \right\}$ (8)

Q.9 a. Show that $L^{-1} \left\{ \frac{S^2}{S^4 + 4a^4} \right\} = \frac{1}{2a} (\cosh at \cdot \sin at + \sinh at \cdot \cos at)$ (8)

b. Evaluate $L^{-1} \left\{ \log \frac{s+1}{s-1} \right\}$ (8)