ROLL NO. _____

Code: DE51/DC51 **Subject: ENGINEERING MATHEMATICS - I**

DipIETE – ET/CS

Time: 3 Hours **DECEMBER 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\lim_{x\to 0} \frac{\sin x^0}{x}$

(A) $\frac{180}{\pi}$ **(B)** $\frac{\pi}{360}$

(C) $\frac{360}{\pi}$ **(D)** $\frac{\pi}{180}$

b. If $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$ then $\frac{dy}{dx}$ is

(A) $-\frac{1}{2}$ **(B)** $\frac{1}{2}$

(C) 1 **(D)** –1

c. $\int \cos^3 x dx$ is

(A) $\frac{\cos 3x}{12}+\frac{3}{4}\cos x + C$ **(B)** $\frac{\sin 3x}{12}-\frac{3}{4}\sin x + C$

(C) $\frac{\sin 3x}{12}+\frac{3}{4}\sin x + C$ **(D)** $\frac{\cos 3x}{12}-\frac{3}{4}\cos x + C$

d. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 4A + 7I$ is

(A) $\begin{bmatrix} 1 & 6 \\ 2 & 4 \end{bmatrix}$ **(B)** $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 8 \\ 0 & 0 \end{bmatrix}$ **(D)** $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

e. If $\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ and ω is a cube root of unity, then the value of Δ is:

(A) –1 **(B)** 3
(C) 0 **(D)** 2

f. The value of $\cos 52^\circ + \cos 68^\circ + \cos 172^\circ$ is

(A) -1 **(B)** 0
(C) 2 **(D)** 1

g. The term independent of x in the expansion of $\left(2x^2 - \frac{1}{x}\right)^{12}$ is

(A) 7820 **(B)** 4595
(C) 8410 **(D)** 7920

h. If cos(A–B)=3cos(A+B), then cot A cot B is:

(A) –2 **(B)** 2
(C) 1 **(D)** 3

i. The area of a triangle whose vertices are (6, 3), (–3, 5) and (4, –2) is:

(A) 24.5 sq units **(B)** 23 sq units
(C) 24 sq units **(D)** 26.4 sq units

j. The equation of the line which makes intercepts –4 and 5 on the axis is:

(A) 4x + 5y + 20 = 0 **(B)** 4x – 5y + 20 = 0
(C) 5x – 4y + 20 = 0 **(D)** 5x + 4y + 20 = 0

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Prove that $\cos\alpha + \cos\left(\alpha + \frac{2\pi}{3}\right) + \cos\left(\alpha + \frac{4\pi}{3}\right) = 0$ **(8)**

b. Prove that $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$ **(8)**

Q.3 a. Find the coefficient of x^{18} in the expansion of $\left(x^2 + \frac{3a}{x}\right)^{15}$ **(8)**

b. If the first term of an A.P is 2 and the sum of first five terms is equal to one fourth of the sum of the next five terms, then (i) show that $T_{20} = -112$ (ii) find the sum of first 30 terms. **(8)**

Q.4 a. Show that

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c) \quad \textbf{(8)}$$

b. Using determinants solve the following system of equations: **(8)**

$$2y - 3z = 0$$
$$x + 3y = -4$$
$$3x + 4y = 3$$

Q.5 a. Find the equation of the right bisector of the segment joining A(1, 1) and B(2, 3) **(8)**

b. Find the equation of the lines through the origin and making an angle of 60° with the line $x + y\sqrt{3} + 3\sqrt{3} = 0$ **(8)**

Q.6 a. Find the equation of the circle which passes through the points (5, –8), (2, –9) and (2, 1). Find also the co-ordinates of its centre and radius. **(8)**

b. Find the length of major and minor axis, eccentricity, the co-ordinates of vertices and foci, directrices and the length latus rectum of the ellipse $3x^2+2y^2=6$. **(8)**

Q.7 a. If $y = \ell og(x + \sqrt{1+x^2})$, Prove that

$$(1 + x^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = 0 \quad \textbf{(8)}$$

b. Find the equation of the tangent to the curve $y = \dfrac{x-7}{(x-2)(x-3)}$ at the point where it cuts x-axis. **(8)**

Q.8 a. Evaluate $\int \dfrac{4x+1}{x^2+3x+2}dx$ **(8)**

b. Evaluate $\displaystyle\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$ **(8)**

Q.9 a. Solve the initial value problem

$$x\frac{dy}{dx} + \cot y = 0, \text{ when } y(\sqrt{2}) = \pi/4$$ **(8)**

b. Solve $(x^2 + xy)dy = (x^2 + y^2)dx$ **(8)**