ROLL NO. _____

Code: AE57/AC57/AT57 Subject: SIGNALS AND SYSTEMS

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The function $\sin(\pi u)/(\pi u)$ is denoted by

(A) $\text{sinc}(\pi u)$ **(B)** $\text{sinc}(u)$
(C) signum **(D)** none of these

b. A periodic signal $x(n)$ of period N_1 is added to another periodic signal of period N_2. The period of the resulting signal is always

(A) $N_1 + N_2$ **(B)** $N_1 N_2$
(C) LCM of N_1 and N_2 **(D)** GCD of N_1 and N_2

c. The unit step response of an LTI system with impulse response $h(n) = \delta(n) - \delta(n-1)$ is

(A) $\delta(n-1)$ **(B)** $\delta(n)$
(C) $u(n-1)$ **(D)** $u(n)$

d. If the fourier series coefficients of a signal are periodic, then the signal must be

(A) continuous-time, periodic **(B)** discrete-time, periodic
(C) continuous-time, nonperiodic **(D)** discrete-time, nonperiodic

e. The Fourier series representations are based on using

(A) constant coefficients **(B)** only cosine functions
(C) only sine functions **(D)** orthogonal functions

f. Let $X[k]$ represents the Discrete-time Fourier series (DTFS) coefficients of the periodic sequence $x(n)$ with period N. The DTFS coefficients of the signal $(-1)^n x(n)$ in terms of $X[k]$ are

(A) $X[k]$ **(B)** $X[-k]$
(C) $X\left[k+\frac{N}{2}\right]$ **(D)** $X\left[k-\frac{N}{2}\right]$

g. The property of Fourier transform that states that the compression in time domain is equivalent to expansion in the frequency domain is

(A) duality **(B)** time shifting
(C) time scaling **(D)** frequency shifting

h. Two signals $x_1(n)$ and $x_2(n)$ are related by $x_2(n) = x_1(-n)$. In the z-domain, their ROCs are

(A) the same **(B)** reciprocal of each other
(C) negative of each other **(D)** complement of each other

i. For distortionless transmission through an LTI system of frequency response $H(\omega)$, the phase of $H(\omega)$ is

(A) constant **(B)** zero
(C) independent of ω **(D)** linearly dependent on ω

j. A system characterized by the system function $H(z) = \frac{1}{2}(1 + z^{-1})$ is a

(A) lowpass filter **(B)** highpass filter
(C) bandpass filter **(D)** bandreject filter

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. For an energy signal $x(t)$ with energy E_x, determine the energy of the following signals:

(i) $x(t - T)$ (ii) $x(at)$
(iii) $x(at - b)$ (iv) $ax(t)$ **(8)**

b. If $x(t) * h(t) = y(t)$, then show that $x(at) * h(at) = \frac{1}{|a|} y(at)$ **(8)**

Q.3 a. Let $X[k]$ represent the DTFS coefficients of the periodic sequence $x(n)$ with period N. Find the DTFS coefficients of $(-1)^n x(n)$ **(6)**

b. Suppose we are given the following information about a periodic signal $x(n)$ with period $N = 8$ and Fourier series coefficients $X[k]$:

(i) $X[k] = -X[k - 4]$ (ii) $x(2n + 1) = (-1)^n$
Sketch one period of $x(n)$ **(10)**

Q.4 a. Given that $x(t)$ has the Fourier transform $X(\omega)$, express the Fourier transforms of the signal listed below in terms of $X(\omega)$.

(i) $x_1(t) = x(1 - t) + x(-1 - t)$ (ii) $x_2(t) = x(3t - 6)$ **(8)**

b. Find the Fourier transform $G(\omega)$ of the signal $g(t) = \frac{1}{\pi t}$ **(8)**

Q.5 a. Given that $x(n)$ has the Fourier transform $X(e^{j\omega})$, express the Fourier transforms of the following signals in terms of $X(e^{j\omega})$.

(i) $x_1(n) = (n - 1)^2 x(n)$ (ii) $x_2(n) = e^{jn\pi/2} x(n + 2)$ **(8)**

b. Let the sequence $x(n)$ be a real sequence and let $X(e^{j\omega}) = DTFT[x(n)]$

(i) Prove that the magnitude spectrum is an even function, that is, $\left|X(e^{j\omega})\right| = \left|X(e^{-j\omega})\right|$

(ii) Prove that the phase spectrum is an odd function, that is, $\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$ **(8)**

Q.6 a. A waveform $x(t) = 10 + 10\sin(500t)$ is to be sampled periodically and reproduced from these samples. Find the maximum allowable time interval between sample values. How many sample values are required to be stored in order to produce 2 seconds of this waveform? **(6)**

b. A signal $x(t) = \sin(\pi t)/(\pi t)$ is sampled by $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - n/2)$. Determine and sketch the sampled signal and its Fourier transform. **(10)**

Q.7 a. Show that for an LTI system, when the input is $x(t) = e^{s_o t}u(t)$, the output is of the form $y(t) = H(s_o)e^{s_o t}u(t)$. How is $H(s_o)$ related to the impulse response of the system? **(6)**

b. Determine the impulse response $h(t)$ of a system having a double-order pole at $s = -a$ and a zero at $s = -b$, where $a, b > 0$ and $b - a = B$. It is also given that $h(0) = 2$ **(10)**

Q.8 a. Apply the final-value theorem of z-transform to determine $x(\infty)$ for the signal

$$x(n) = \begin{cases} 1, & \text{if } n \text{ is even} \\ 0, & \text{otherwise} \end{cases}$$ **(7)**

b. An LTI system is characterized by the system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5z^{-1} + 1.5z^{-2}}$$

Specify the ROC of $H(z)$ and determine the impulse response $h(n)$ for the following conditions:

(i) The system is causal and unstable

(ii) The system is noncausal and stable

(iii) The system is anticausal and unstable **(9)**

Q.9 a. Define the terms mean, variance, co-variance and correlation coefficient as applied to random variable X with pdf $f_X(x)$. **(8)**

b. Find the power spectral density for the random process $X(t) = 4\cos(5\pi t)$ and also compute the power in the random process. **(8)**