ROLL NO. ________________

Code: AE56/AC56/AT56 Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2013** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $f(z)=u(x,y)+iv(x,y)$ is analytic, then its derivative $f'(z)$ is equal to

(A) $\frac{\partial v}{\partial y}+i\frac{\partial v}{\partial x}$ **(B)** $\frac{\partial u}{\partial x}+i\frac{\partial v}{\partial y}$

(C) $\frac{\partial u}{\partial x}+i\frac{\partial u}{\partial y}$ **(D)** None of these

b. The value of the integral $\int_C \frac{2z+1}{z^2+z}dz$, where C is $|z|=\frac{1}{2}$, is

(A) πi **(B)** $2\pi i$
(C) $4\pi i$ **(D)** 0

c. A unit tangent vector to the curve $x=t^2+2, y=4t-5, z=2t^2-6t$, at the point t = 2, is

(A) $\frac{6i+3j+4k}{\sqrt{61}}$ **(B)** $\frac{6i+3j-4k}{\sqrt{61}}$

(C) $2i+2j+k$ **(D)** $\frac{2i+2j+k}{3}$

d. The value of the line integral $\int_C \left[\left(5xy-6x^2\right)dx+\left(2y-4x\right)dy\right]$ along the curve $y=x^3$ from the point (1,1) to (2, 8) is

(A) 15 **(B)** 28
(C) 35 **(D)** 40

e. The value of $\Delta^3[(1+x)(1+2x)(1+3x)]$, if interval of differencing is 1, is

(A) 36 **(B)** 24
(C) 18 **(D)** 6

f. If $y_1 = 1, y_3 = 4, y_4 = 8$, then y_2 is equal to

(A) 2 **(B)** $\frac{5}{3}$
(C) $\frac{5}{4}$ **(D)** 3

g. The differential equation of a family of spheres $x^2 + y^2 + (z-c)^2 = R^2$, is

(A) $xp + yq = 0$ **(B)** $xp - yq = 0$
(C) $yp + xq = 0$ **(D)** $yp - xq = 0$

h. A party of n persons take their seats at random at a round table. The probability that two specified persons always sit together is

(A) $\frac{2}{n}$ **(B)** $\frac{2}{n-1}$
(C) $\frac{2}{n-2}$ **(D)** None of these

i. If the diameter of an electric cable is assumed to a continuous variate with p.d.f. $f(x) = kx(1-x), 0 \le x \le 1$, then K is equal to

(A) 4 **(B)** 5
(C) 6 **(D)** 8

j. If a random variable has a Poisson distribution such that 2P(1) = P(2), then the variance is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof. **(8)**

b. Find the bilinear transformation which maps the points $i, -i, 1$ of z-plane into $0, 1, \infty$ of w-plane respectively. **(8)**

Q.3 a. Find Laurent's series expansion of $\frac{z^2-1}{z^2+5z+6}$ about z=0 in the region $2 < |z| < 3$. **(8)**

b. Use Residue theorem to evaluate $\int_C \frac{1-2z}{z(z-1)(z-2)} dz, C: |z| = 1.5$ **(8)**

Q.4 a. If $u = x^2 + y^2 + z^2$ and $V = xI + yJ + zK$, show that $\text{div}(uV) = 5u$ **(8)**

b. Find the angle between the normals to the surface $xy = z^2$ at the points (4, 1, 2) and $(3,3,-3)$. **(8)**

Q.5 a. Apply Green's theorem to evaluate $\int_C \left[\left(3x - 8y^2\right)dx + (4y - 6xy)dy\right]$

Where C is the boundary of the region bounded by x=0, y=0, x+y=1 **(8)**

b. Use Divergence theorem to evaluate $\iint_S \left[x^3 dydz + x^2 ydzdx + x^2 zdxdy\right]$ where S is the closed surface consisting of the cylinder $x^2 + y^2 = a^2$ and the circular discs z=0 and z=2. **(8)**

Q.6 a. Use Newton's divided difference formula to evaluate f(8) given that **(8)**

X	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

b. Find an approximate value of $\log_e 5$ by calculating to four decimal places, by Simpson's $\frac{1}{3}$rd rule,

$$\int_0^5 \frac{dx}{4x+5}$$

dividing the range into ten equal parts. **(8)**

Q.7 a. Apply Charpit's method to solve $\left(a^2 + b^2\right)y = bz$. **(8)**

b. Use method of separation of variables to solve $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$, given that $u(0, y) = 8e^{-3y}$. **(8)**

Q.8 a. A committee consists of 9 students two of which are from 1st year, three from 2nd year and four from 3rd year. Three students are to be removed at random. What is the chance that
(i) the three students belong to different classes.
(ii) two belong to the same class and third to the different class. **(8)**

b. In a certain college, 4% of the boys and 1% of girls are taller than 1.8m. Moreover 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl? **(8)**

Q.9 a. Fit a Poisson distribution to the set of observations: **(8)**

x	0	1	2	3	4
f	122	60	15	2	1

b. Assuming that the diameters of 1000 brass plugs taken consecutively from a machine, form a normal distribution with mean 0.7515 cm and standard deviation 0.0020 cm, how many of the plugs are likely to be rejected if the approved diameter is $0.752 \pm 0.004\text{cm}$? (Given: if z is the normal variable, then area under normal curve for $0 \le z \le 1.75$ is 0.4599 and for $0 \le z \le 2.25$ is 0.4878.) **(8)**