Code: AE51/AC51/AT51 **Subject: ENGINEERING MATHEMATICS - I**

AMIETE – ET/CS/IT

Time: 3 Hours **DECEMBER 2012** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions, answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $u = \dfrac{x(x^3 - y^3)}{x^3 + y^3}$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

(A) 0 **(B)** u

(C) 2u **(D)** $\dfrac{1}{2}u$

b. If $x = r\cos\theta, y = r\sin\theta$ then the value of $\dfrac{\partial r}{\partial x}$ is equal to

(A) 1 **(B)** x

(C) $\dfrac{x}{r}$ **(D)** $\dfrac{r}{x}$

c. The value of integral $\int_0^1 dx \int_0^x e^{y/x} dy$ is equal to

(A) $\dfrac{1}{2}$ **(B)** e^2

(C) $\dfrac{1}{2}(e-1)$ **(D)** $\dfrac{1}{4}(e-1)$

d. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 2 & 4 & 6 \end{bmatrix}$ is equal to

(A) 0 **(B)** 2
(C) 3 **(D)** does not exist

e. Using Newton-Raphson method to root of the equation f(x)=0 fails if

(A) f(x) is an exponential function **(B)** $f'(x)$ is zero
(C) $|f'(x)|=1$ **(D)** None of these

f. The degree of the differential equation $x^2\left(\frac{d^2y}{dx^2}\right)^3 + y\left(\frac{dy}{dx}\right)^4 + y^4 = 0$ is equal to

(A) 1 **(B)** 2
(C) 3 **(D)** 4

g. Integrating factor of the differential equation $(x+1)\frac{dy}{dx}=y+e^x(x+1)^2$ is equal to

(A) e^x **(B)** e^{x+1}
(C) $\frac{1}{x+1}$ **(D)** x+1

h. Particular Integral (PI) for the differential equation $(D^2+4)y=\sin 3x$ is equal to

(A) sin 3x **(B)** cos 3x
(C) $\frac{1}{5}\sin 3x$ **(D)** $-\frac{1}{5}\sin 3x$

i. The value of the integral $\int_0^\infty x^{1/4}e^{-\sqrt{x}}dx$ is equal to

(A) $\frac{3}{2}\sqrt{\pi}$ **(B)** $\sqrt{\pi}$
(C) $\frac{5}{2}\sqrt{\pi}$ **(D)** $\frac{1}{2}\sqrt{\pi}$

j. The value of the integral $\int_{-1}^{+1}P_m(x).P_n(x)dx,\ n \neq m$ is equal to

(A) 1 **(B)** –1
(C) 0 **(D)** Does not exists

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If x+y+z=u, y+z=uv, z=uvw then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$ **(8)**

b. Find the stationary values of $x^2+y^2+z^2$ subject to the conditions $ax^2+by^2+cz^2=1$ and lx+my+nz=0 **(8)**

Q.3 a. Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2 + y^2)\,dy\,dx$ **(8)**

b. Compute $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ if the region of integration is bounded by the co-ordinate planes and the plane x+y+z=1 **(8)**

Q.4 a. Determine for what values of λ and μ the following equations have
(i) no solution
(ii) an unique solution
(iii) infinite number of solutions

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$
(8)

b. Find the eigenvalues and the corresponding eigenvectors for the matrix

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$
(8)

Q.5 a. Write the Newton-Raphson procedure for finding $\sqrt[3]{N}$ where N is a real number. Use it to find $\sqrt[3]{18}$ correct to 2 decimals, assuming 2.5 as the initial approximation. **(8)**

b. Solve the following system of equations using Gauss-Seidal method

$$6x + y + z = 105$$
$$4x + 8y + 3z = 155$$
$$5x + 4y - 10z = 65$$
(Perform four iterations) **(8)**

Q.6 a. Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$ **(8)**

b. Solve the equation $\left[(\cos x)\log_e(2y-8) + \frac{1}{x}\right]dx + \frac{\sin x}{y-4}dy = 0$ **(8)**

Q.7 a. Solve $x^2 \frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 \log x$ **(8)**

b. Using the method of variation of parameters, solve $\frac{d^2y}{dx^2} + 4y = 4\tan 2x$ **(8)**

Q.8 a. Show that $\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right) = 2\int_0^{\pi/2}\sqrt{\tan\theta}\,d\theta = 4\int_0^{\infty}\frac{x^2dx}{1+x^4} = \pi\sqrt{2}$ **(8)**

b. Find solution in generalized series form about x=0 of the differential equation $3x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ **(8)**

Q.9 a. Prove that $J_n(x) = \frac{x}{2n}\left[J_{n-1}(x) + J_{n+1}(x)\right]$ **(8)**

b. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ in terms of Legendre Polynomials. **(8)**