

## AMIETE – CS

Time: 3 Hours

**DECEMBER 2012**

Max. Marks: 100

*PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.*

**NOTE:** There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

**Q.1 Choose the correct or the best alternative in the following: (2×10)**

- a. Number of elements in the power set of set  $A = \{\phi\}$  is
- (A) 0 (B) 2  
(C) 1 (D) 4
- b. Number of relations that can be defined on set  $A = \{1, 2, 3, 4\}$  is
- (A) 1024 (B) 16  
(C) 65536 (D) 256
- c. Number of ways in which the letters of the word BANANA can be distinctly arranged is
- (A) 60 (B) 720  
(C) 16 (D) 120
- d. Which of the following is not a rational number
- (A) Sum of two rational numbers (B) Product of two irrational numbers  
(C) Sum of two recurring numbers (D) Difference of two rational numbers
- e. Suppose the universe consists of all integers. Consider  
 $P(x): x \leq 3,$   
 $r(x): x > 0$   
 What is the truth value of  $\neg P(3) \vee r(0)$ ?
- (A) TRUE (B) FALSE
- f. Relation defined on a partially ordered set is not
- (A) Anti-Symmetric (B) Symmetric  
(C) Transitive (D) Reflexive
- g. Which of the following is not true about composition of functions?
- (A) Associative (B) Commutative  
(C) Existence of Inverse (D) Existence of Identity

**Code: AC65      Subject: DISCRETE STRUCTURES**

- h. A group G is cyclic if
- (A) G is a subgroup of a cyclic group
  - (B) G is of finite order
  - (C) G is isomorphic to a cyclic group
  - (D) None of these
- i. The hamming distance between  $x = 111000111$  and  $y = 000100111$  is
- (A) 3
  - (B) 4
  - (C) 5
  - (D) 6
- j. Addition of two integers x and y is even then
- (A) Both are odd
  - (B) x is even and y is odd
  - (C) x is odd and y is even
  - (D) x is odd and y is even multiple of x

**Answer any FIVE Questions out of EIGHT Questions.  
Each question carries 16 marks.**

- Q.2** a. Using Venn diagrams, prove that, for any sets A, B, C.
- $$(A \cup B) \cap C = (\bar{A} \cap \bar{B}) \cup \bar{C} \quad (8)$$
- b. Three students X, Y, Z write an examination. Their chances of passing are  $\frac{1}{2}$ ,  $\frac{1}{3}$  and  $\frac{1}{4}$  respectively. Find the probability that
- (i) all of them pass
  - (ii) at least one of them passes
  - (iii) at least two of them pass. (8)
- Q.3** a. Read the following paragraph. Convert each statement into propositional expressions and show that the conclusion is true.
- “Ram is a student and he is sincere. Every sincere student excels in the class. Shyam is also a sincere student. Therefore Ram and Shyam both excel in their class.” (8)
- b. Show that the Boolean expressions  $(z' \vee x) \wedge ((x \wedge y) \vee z) \wedge (z' \vee y)$  and  $x \wedge y$  are logically equivalent. (8)
- Q.4** a. Prove that  $R \rightarrow S$  is a valid conclusion from the premises  $P \rightarrow (Q \rightarrow S)$ ,  $\sim R \vee P$  and Q. (8)
- b. Prove that
- $$\{\forall x [P(x) \vee Q(x)]\} \wedge \{\exists x \neg P(x)\} \wedge \{\forall x [\neg Q(x) \vee R(x)]\} \wedge \{\forall x [S(x) \rightarrow \neg R(x)]\} \Rightarrow \exists x \neg S(x) \quad (8)$$
- Q.5** a. Using mathematical induction method, prove that  $11^{n+2} + 12^{2n+1}$  is divisible by 133 for any  $n \in \mathbb{N}$ . (8)

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- b. A sequence  $\{a_n\}$  is defined recursively by  $a_1 = 4$ ,  $a_n = a_{n-1} + n$  for  $n \geq 2$ . Find  $a_n$  in explicit form. (8)
- Q.6** a. Show that if a relation  $R$  defined on a set  $A$  is symmetric and transitive then  $R$  is not irreflexive. (8)
- b. Let  $L$  is a distributive lattice. Show that if there exists an element  $a$  such that  $a \wedge x = a \wedge y$  and  $a \vee x = a \vee y$  then  $x = y$ . (8)
- Q.7** a. Let  $f$  be a function defined on a real number  $R$  such that  $f(x) = 2x - 1$  then show that  $f$  is an invertible function and find inverse function of  $f$ . (8)
- b. Define even permutation function. Show that composition of two even permutation function is even permutation function. (8)
- Q.8** a. Let  $(G, *)$  be a group then for any two elements  $a$  and  $b$  of  $(G, *)$  prove that  $(a * b)^{-1} = b^{-1} * a^{-1}$  (8)
- b. Show that set  $G = \{1, i, -i, -1\}$  forms a cyclic group under the binary operation of multiplication. (8)
- Q.9** Write short notes on **TWO** of the followings:
- (i) Parity Check Matrix
  - (ii) Hamming distance
  - (iii) Ring  $z_n$
- (8+8)