ROLL NO. ___________________

Code: DE105/DC105 **Subject: ENGINEERING MATHEMATICS II**

DipIETE – ET/CS (New Scheme)

Time: 3 Hours **DECEMBER 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. $\lim_{x\to 0}\left\{\frac{e^x+\sin x-1}{\log(1+x)}\right\}$ is equal to

(A) 0 **(B)** 2
(C) –2 **(D)** 1/2

b. The value of $\int_0^{\pi/2} \sin^6 x \cos^3 x\, dx$ is equal to

(A) $\frac{5\pi}{4096}$ **(B)** $\frac{5\pi}{4069}$
(C) $\frac{5\pi}{6094}$ **(D)** $\frac{5\pi}{9064}$

c. If $Z_1 = 2-5i,\ Z_2 = -1+4i,\ Z_3 = 6+i$ and $Z_4 = 3-7i$, then $\frac{(Z_1+Z_2)Z_3}{Z_4}$ equals

(A) $\frac{28}{29}-\frac{17}{29}i$ **(B)** $\frac{-28}{29}+\frac{17}{29}i$
(C) $\frac{-28}{29}-\frac{17}{29}i$ **(D)** $\frac{28}{29}+\frac{17}{29}i$

d. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ and are such that $\vec{a}+\vec{b}$ is a unit vector, then θ is equal to

(A) $\frac{\pi}{4}$ **(B)** $\frac{\pi}{3}$
(C) $\frac{\pi}{2}$ **(D)** $\frac{2\pi}{3}$

e. Let $\vec{a}$ = (1,2,0), $\vec{b}$ = (–3,2,0), $\vec{c}$ = (2,3,4) then $\vec{a}\cdot(\vec{b}\times\vec{c})$ equal

(A) 31 **(B)** 30
(C) 33 **(D)** 32

f. The solution of the differential equation $(D^2+4)y = e^x$ is

(A) $C_1 \cos 2x - C_2 \sin 2x + \frac{e^x}{4}$ **(B)** $C_1 \cos 2x + C_2 \sin 2x + \frac{e^x}{4}$

(C) $C_1 \cos 2x + C_2 \sin 2x + \frac{e^x}{5}$ **(D)** $C_1 \cos 2x - C_2 \sin 2x + \frac{e^x}{5}$

g. The voltage and current of a circuit are given by the complex numbers $3+4j$ and $2-5j$ respectively. The complex number that will be the impedance of the circuit is

(A) $\frac{14}{29}+\frac{23}{29}j$ **(B)** $\frac{14}{29}-\frac{23}{29}j$

(C) $\frac{-14}{29}-\frac{23}{29}j$ **(D)** $\frac{-14}{29}+\frac{23}{29}j$

h. The series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots\infty$ is

(A) Convergent **(B)** Divergent

(C) Oscillatory **(D)** None of these

i. The Laplace transform of $4\sin^3 t$ is

(A) $\frac{s}{s^2+1}-\frac{3}{s^2+9}$ **(B)** $\frac{3}{s^2+1}-\frac{3}{s^2+9}$

(C) $\frac{1}{s^2+1}+\frac{s}{s^2+9}$ **(D)** $\frac{1}{s^2+1}+\frac{1}{s^2+9}$

j. $L^{-1}\left(\frac{1}{s(s^2+1)}\right)$ is

(A) $1+\cos t$ **(B)** $1-\sin t$

(C) $1-\cos t$ **(D)** $1+\sin t$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to 4 places of decimal. **(8)**

b. Evaluate $\lim\limits_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ **(8)**

Q.3 a. Evaluate by using reduction formula **(8)**
$\int_0^{\pi/6} \cos^4 3\phi \sin^2 6\phi \, d\phi$

b. Find the area common to the parabola $y^2 = x$ and the circle $x^2 + y^2 = 2$. **(8)**

Q.4 a. If n is positive integer, prove that

$\left(\sqrt{3}+i\right)^n + \left(\sqrt{3}-i\right)^n = 2^{n+1} \cos \frac{n\pi}{6}, \ \left(i = \sqrt{-1}\right)$ **(8)**

b. Two impedances $Z_1 = 8 + j6$ ohms & $Z_2 = 6 - j8$ ohms are connected in parallel across 200 volts, calculate the magnitude of current in each branch and the total current in the circuit. **(8)**

Q.5 a. Forces of magnitude 5 and 3 units acting in the directions $6i + 2j + 3k$ and $3i - 2j + 6k$ respectively act on a particle which is displaced from the point (2,2,-1) to (4,3,1). Find the work done by the forces. **(8)**

b. Find the volume of the parallelepiped, if $\vec{a} = -3i + 7j - 5k$, $\vec{b} = -3i + 7j - 3k$ and $\vec{c} = 7i - 5j - 3k$ are the three coterminous edges of the parallelepiped. **(8)**

Q.6 a. Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin 2x$ **(8)**

b. A Condenser of capacity C is discharged through the inductance L and a resistance R in series and the charge q at any time t satisfies the equation

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0$$

Given that L = 0.25 henry, R = 250 ohms, $C = 2\times10^{-6}$ farad and that when t = 0, the charge q=0.002 coulombs, and the current $\frac{dq}{dt} = 0$, obtain the value of q in terms of t. **(8)**

Q.7 Test for convergence the series given below:

a. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$ **(8)**

b. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$ **(8)**

Q.8 Find the Laplace transform of $f(t)$ where

a. $f(t) = 2 \sin 2t \cos 4t$ **(8)**

b. $\mathrm{f(t)} = \frac{\mathrm{e^{at} - \cos bt}}{\mathrm{t}}$ **(8)**

Q.9 a. Find the inverse Laplace transform of $\frac{s-4}{4(s-3)^2 + 16}$ **(8)**

b. Apply convolution theorem to find $L^{-1} = \left\{ \frac{s}{(s^2+1)(s^2+4)} \right\}$ **(8)**