Code: CT22 **Subject: DISCRETE MATHEMATICAL STRUCTURES**

ALCCS

Time: 3 Hours **DECEMBER 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE:

- **Question 1 is compulsory and carries 28 marks. Answer any FOUR questions from the rest. Marks are indicated against each question.**
- **Parts of a question should be answered at the same place.**

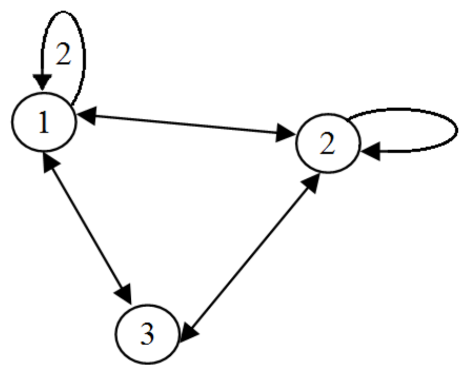
Q.1 a. Show that the truth values of the following compound proposition is independent of the truth values of their components

$\{p \wedge (p \to q)\} \to q$

b. If A,B,C are finite sets, prove the extended addition principle

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

c The digraph of a relation R on the set {1,2,3} is as given below. Determine whether R is an equivalence relation or not?



d. Draw the Hasse diagram representing the positive divisors of 36.

e. State and prove Demorgan's law in Boolean algebra B.

f. Give an example of a graph that has

(i) An Euler Circuit but no Hamiltonian cycle

(ii)An Hamiltonian cycle but no Euler circuit

g. Give a regular expression for the language of all strings in $\{0,1,2\}^*$ containing exactly two 2's. **(7×4)**

Q.2 a. Obtain PDNF of following: **(5)**

$(P \vee (R \to \sim (Q \vee P))) \wedge R$ without using truth table.

b. Show that Q is a valid conclusion for the premises: **(5)**
$\sim S$, $P \vee (Q \wedge R)$, $P \leftrightarrow S$

c. Let A be any set and P(A) be the power set of A. Show that it is a lattice under the partial order defined as $X \leq Y \leftrightarrow X \subseteq Y$. **(8)**

Q.3 a. Show that if any 5 numbers are chosen from {1 to 8}, then two of them will add upto 9. **(6)**

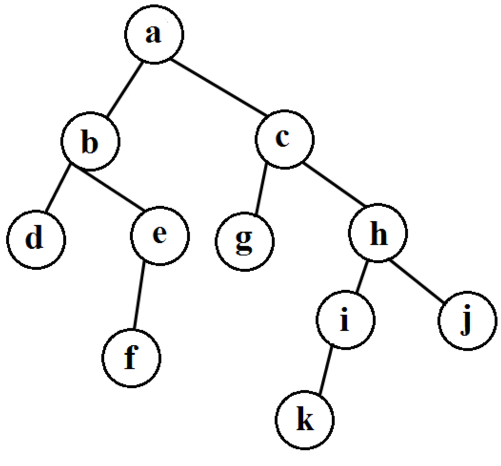
b. Show that: **(6)**
$\forall P(x) \wedge \exists Q(x) \Rightarrow \exists(P(X) \wedge Q(X))$

c. Let $f : R \rightarrow R$ be defined as **(6)**
$f(x) = 2x - 3$ if $x \leq 9$
$= x^2 - 4x + 7$, if $9 < x < 100$
$= \cos \Pi x$ if $100 \leq x$
Find f(7), f(8), f(9), f(10) and f (100)

Q.4 a. If L1 and L2 are regular languages ever Σ show that $L1 \cup L2$ is regular. **(6)**

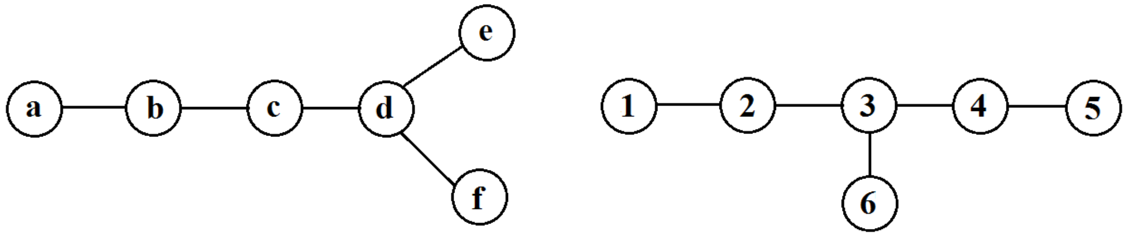
b. Construct a finite automata to accept all the strings of odd lengths on the alphabet {a,b,c}.
(6)

c. Traverse the tree using
(i) Preorder traversal
(ii) Inorder traversal
(iii) Post order traversal **(6)**

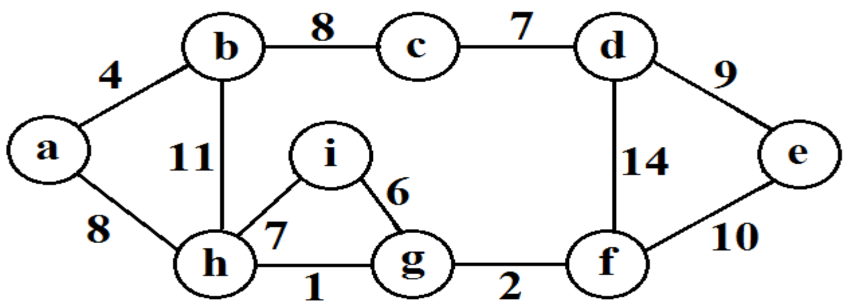


Q.5 a. In a tree (with two or more vertices), prove that there are at least two pendant vertices. **(5)**

b. Define isomorphism of graphs. Verify following graphs for isomorphism. **(5)**



c. Define a spanning tree of a graph. Does every graph have a spanning tree? Find the minimum spanning tree of the following graph using Kruskal's algorithm. **(8)**



Q.6 a. Explain how Binary Search method fails to find 43 in the following given sorted array: **(6)**
8, 12, 25, 26, 35, 48, 57, 78, 86, 93, 97, 108, 135, 168, 201

b. How can the output of the Floyd-Warshall algorithm be used to detect the presence of a negative-weight cycle? **(6)**

c. Define Cartesian product on sets. For the given sets X = {1, 2}, Y = {a, b, c} and Z = {c, d}, find $(X\times Y)\cap(X\times Z)$. **(6)**

Q.7 a. Draw the ordered rooted tree that represent the expression $((x+y)\uparrow 2)+((x-4)/3)$. How do you find the equivalent prefix and postfix expressions from the tree? **(6)**

b. Prove that deterministic and nondeterministic finite automata are equivalent. **(6)**

c. Define regular expression. (i) Let $L = \{w\in \{a, b\}^* : |w| \equiv_3 0\}$. List the first six elements in a lexicographic enumeration of L. (ii) $L = \{w\in \{a, b\}^*$: all prefixes of w end in a$\}$. List the elements of L. **(6)**