ROLL NO. ________________

Code: AE56/AC56/AT56/AE107/AC107/AT107 Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **DECEMBER 2015** Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following : (2×10)

a. The harmonic conjugate of $u(x, y) = 4xy + x + 1$ is :

(A) $2y^2 + y - 2x^2 + c$ **(B)** $2x^2 - y^2 + y + c$

(C) $2y^2 - y + c$ **(D)** $y^2 - x^2 + c$

b. If $f(z)$ is analytic within and on a simple closed curve *C* and *a* is any point inside *C*, then $f(a)$ is :

(A) 0 **(B)** $\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)}dz$

(C) $\frac{1}{\pi i}\int_C \frac{f(z)}{(z-a)}dz$ **(D)** $\frac{1}{2\pi i}\int_C \frac{f(z)}{(z-a)^2}dz$

c. A velocity vector is given as $= 5xyi + 2y^2j + 3yz^2k$, the divergence of this velocity vector at (1 , 1 , 1) is :

(A) 9 **(B)** 10

(C) 14 **(D)** 15

d. Stoke's theorem connects :

(A) A line integral and a surface integral
(B) A surface integral and a volume integral
(C) A line integral and a volume integral
(D) Gradient of a function and its surface integral

e. The work done by the force $F = (5xy - 6x^2)i + (2y - 4x)j$ in moving a particle along the curve $y = x^3$ from the point (1,1) to (2,8) is :

(A) 25 **(B)** 15

(C) 35 **(D)** 45

f. Eliminating a and b from $z = ae^{bt}\sin bx$, we obtain the partial differential equation :

(A) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial t^2}$ **(B)** $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial t}$

(C) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial z}{\partial t}$ **(D)** $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial t^2} = 0$

g. Which of the following relation amongst finite differences is not correct :

(A) $\Delta^3 y_{-1} - \Delta^3 y_{-2} = \Delta^4 y_{-2}$ **(B)** $\Delta^3 y_{-1} - \Delta^3 y_0 = \Delta^4 y_0$

(C) $\Delta y_2 - \Delta y_1 = \Delta^2 y_1$ **(D)** $y_2 - y_1 = \Delta y_1$

h. Two random variables X and Y are said to be independent if :

(A) $E(XY) = 1$ **(B)** $E(XY) = 0$

(C) $E(XY) = E(X)\,E(Y)$ **(D)** $E(XY) =$ any constant value

i. To apply Simpson's 1/3rd and 3/8th rules both, the interval must be divided into minimum of sub-interval are :

(A) 4 **(B)** 5

(C) 6 **(D)** 8

j. A box contains 12 items out of which 4 are defective. A person selects 6 items from the box. The expected number of defective items out of his selected items is :

(A) 2 **(B)** 3

(C) 3/2 **(D)** ½

Answer any FIVE questions out of EIGHT questions.
Each question carries 16 marks.

Q.2 a. Use Cauchy_Riemann equations to find v where $u = 3x^2y - y^3$. **(8)**

b. Show that the transformation $w = \frac{2z+3}{z-4}$ maps the circle $x^2 + y^2 - 4x = 0$ onto the straight line $4u + 3 = 0$, and explain why the curve obtained is not a circle. **(8)**

Q.3 a. Prove that the function $f(z) = u + iv$ where $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$ $(z \neq 0), f(0) = 0$ is continuous and that Cauchy_Riemann equations are satisfied at the origin, yet $f'(z)$ does not exist there. **(8)**

b. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series valid for the regions (i) $|z| < 1$ (ii) $1 < |z| < 3$. **(8)**

Q.4 a. If $(\varepsilon) = \int_C \frac{6z^2+5z+2}{z-\varepsilon}\,dz$, where C is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$, find the values of $F(i), F'(-i)$ and $F''(-1)$. **(8)**

b. If F $= xy^2 i + 2x^2yzj - 3yz^2k$, find (i) Div F (ii) Curl F at the point (1, -1, 1). **(8)**

Q.5 a. Find the directional derivative of $\emptyset = (x^2 + y^2 + z^2)^{\frac{-1}{2}}$ at the point P(3,1,2) in the direction of the vector $yzi + xzj + xyk$. **(8)**

b. Evaluate $\iint_S F.n\,dS$ with the help of Gauss's divergence theorem for $F = 4xi - 2y^2j + z^2k$ taken over the region S bounded by $x^2 + y^2 = 4, z = 0$ and z = 3. **(8)**

Q.6 a Solve the partial differential equation : **(8)**
$(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

b. Solve by Charpit's method : **(8)**
$z = px + qy + pq$

Q.7 a. Using Newton's divided difference formula, find $f(4)$ if : **(8)**

x	:	–4	–1	0	2	5
$f(x)$	:	1245	33	5	9	1335

b. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ by using Simpson's 1/3rd and 3/8th rule. Hence obtain the approximate value of π in each case. **(8)**

Q.8 a. In an examination the number of candidates who secured marks between certain limits were as follows :

Marks	:	0-19	20-39	40-59	60-79	80-99
No. of Candidates	:	41	62	65	50	17

Estimate the number of candidates getting marks less than 70. **(8)**

b. From a pack of 52 cards, 6 cards are drawn at random. Find the probability of the following events : **(8)**
(i) Three are red and 3 are black cards
(ii) Three are kings and 3 are queens

Q.9 a. A continuous random variable X has the density function
$(x) = 3x^2, 0 \le x < 1$.
Find a and b, when
(i) $P(X \le a) = P(X > a)$; (ii) $P(X > b) = 0.05$ **(8)**

b. Fit a Poisson's distribution to the following and calculate theoretical frequencies ($e^{-0.5} = 0.61$) :

Deaths	:	0	1	2	3	4
Frequency	:	122	60	15	2	1

(8)