ROLL NO. ___________________

Code: AE51/AC51/AT51 Subject: ENGINEERING MATHEMATICS-1
AE101/AC101/AT101

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **DECEMBER 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $u = \log\frac{x^2}{y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(A) 0 **(B)** 1
(C) u **(D)** 2u

b. The value of the double integral $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\,dy\,dx$ is

(A) $\frac{a^3}{4}$ **(B)** a^3

(C) $\frac{a^3}{2}$ **(D)** $\frac{a^3}{3}$

c. The rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ is

(A) 1 **(B)** 2

(C) 3 **(D)** 4

d. The Newton-Raphson algorithm for finding cube root of N is

(A) $x_{n+1} = \frac{1}{3}\left[2x_n + \frac{N}{x_n^2}\right]$ **(B)** $x_{n+1} = \frac{1}{3}\left[x_n + \frac{N}{x_n^2}\right]$

(C) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ **(D)** $x_{n+1} = \frac{1}{2}\left[2x_n + \frac{N}{x_n}\right]$

e. The solution of differential equation $(x^2-y^2)\,dx = 2xydy$ is

(A) $x^3-xy^2-x^2y = C$ **(B)** $x^3-3xy^2 = C$
(C) $x^3-xy^2+x^2y = C$ **(D)** $x^3+3xy^2 = C$

f. Particular integral of the differential equation $(D^2-4D+3)y = e^{3x}$ is

(A) $\frac{1}{3}e^{3x}$ **(B)** $\frac{1}{4}e^{3x}$

(C) $\frac{x}{2}e^{3x}$ **(D)** $\frac{1}{3}xe^{3x}$

g. The solution of differential equation $\frac{d^2y}{dx^2} + (a+b)\frac{dy}{dx} + aby = 0$

(A) $y = C_1e^{ax} + C_2e^{-bx}$ **(B)** $y = C_1e^{-ax} + C_2e^{bx}$
(C) $y = C_1e^{ax} + C_2e^{bx}$ **(D)** $y = C_1e^{-ax} + C_2e^{-bx}$

h. $\beta(m+1, n)$ is equal to

(A) $\frac{m}{m+n}\beta(m,n)$ **(B)** $\frac{n}{m+n}\beta(m,n)$

(C) $\frac{m-n}{m+n}\beta(m,n)$ **(D)** $\beta(m,n)$

i. $J^2_{\frac{1}{2}}(x) + J^2_{-\frac{1}{2}}(x)$ is equal to

(A) $\frac{\pi x}{2}$ **(B)** $\frac{2}{\pi x}$

(C) $\frac{\pi}{2x}$ **(D)** $\frac{2x}{\pi}$

j. $5x^3+3x^2+x$ is equal to

(A) $5P_3(x) + 3P_2(x) + P_1(x)$ **(B)** $2P_3(x) + 3P_2(x) + 4P_1(x)+P_0(x)$

(C) $2P_3(x) + 2P_2(x) + 4P_1(x)+P_0(x)$ **(D)** $P_3(x) + P_2(x) + 3P_1(x)+P_0(x)$

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. State and Prove Euler's theorem **(2+6)**

b. If u = x + y + z, uv = y + z, uvw = z, show that **(8)**

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$$

Q.3 a. Change the order of integration and evaluate $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} (x^2 + y^2)\,dx\,dy$ **(8)**

b. Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes y+z = 4 and z = 0 **(8)**

Q.4 a. Investigate for consistency of the following equations and if possible find the solution: 4x -2y + 6z = 8, x + y – 3z = -1, 15x -3y + 9z = 21 **(8)**

b. Find the eigen values and eigen vectors of the matrix. **(8)**

$$\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

Q.5 a. Use Regula-Falsi method to compute a real root of the equation $x \log_{10} x = 1.2$ correct to three decimal. **(8)**

b. Apply Runge-kutta method to find an approximate value of y for x = 0.2 if $\frac{dy}{dx} = x + y^2$ given that y = 1 when x = 0. Take h = 0.2 **(8)**

Q.6 a. Use method of variation of parameters to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$ **(8)**

b. Solve the simultaneous equation: $\frac{dx}{dt} + y = \sin t, \frac{dy}{dt} + x = \cos t$ given that x = 2 and y = 0 when t = 0. **(8)**

Q.7 a. Obtain the series solution of $\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ **(8)**

b. Show that $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx \times \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{\sin x}}\, dx = \prod$ **(8)**

Q.8 a. Prove that $(1 - x^2)\, P_n'(n) = (n+1)[x\, P_n(n) - P_{n+1}(n)]$ **(8)**

b. Show that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} t^n J_n(n)$ **(8)**

Q.9 (For Current Scheme students i.e. AE51/AC51/AT51)

a. Solve the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$ **(8)**

b. Find the orthogonal trajectories of the family of hyperbolas $xy = C^2$ **(8)**

Q.9 (For New Scheme students i.e. AE101/AC101/AT101)

a. Find the Fourier sine transform of $\frac{e^{-ax}}{x}$ **(8)**

b. Find the z-transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$ **(8)**