ROLL NO. ___________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **DECEMBER 2015** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. A relation R is defined on ordered pairs of integers as follows:
$(x, y)R(u, v)$ if $x < u$ and $y > v$. Then R is
(A) Neither a partial order nor an equivalence relation
(B) A partial order but not a total order
(C) A total order
(D) An equivalence relation

b. What is the possible number of reflexive relations on a set of 5 elements?
(A) 2^{10} **(B)** 2^{15}
(C) 2^{20} **(D)** 2^{25}

c. Consider the set $S = \{1, w, w^2\}$ where w and w^2 are cube roots of unity. If * denotes the multiplication operation, the structure $(S,*)$ forms
(A) A group **(B)** A ring
(C) An Integral domain **(D)** A field

d. The number of different ways for n people to arrange themselves in a straight line is
(A) n **(B)** $n!$
(C) n-1 **(D)** n^2

e. Power set P(A) will be having cardinality of
(A) 2 **(B)** 2^2
(C) 2^3 **(D)** $2^{|A|}$

f. Find two incomparable elements in the poset $(\{1,2,4,6,8\},1)$
(A) (1,2) and (2,4) **(B)** (4,6) and (6,6)
(C) (4,6) and (6,8) **(D)** (6,8) and (8,8)

g. The Hamming distance between the codes x = 001100, y = 010110 is
(A) 1 **(B)** 2
(C) 3 **(D)** 4

h. The solution of the recurrence relation $a_n = a.a_{n-1}$ with the initial conditions $a_0 = 1$, $a_1 = a$ is
(A) a^{n-1} **(B)** a^{n-2}
(C) a^{n-3} **(D)** a^n

i. Which one of the following is not necessarily a property of group?
(A) Associativity
(B) Commutativity
(C) Existence of inverse for every statement
(D) Existence of identity

j. The mean and variance are equals in
(A) Binomial distribution **(B)** Poission distribution
(C) Normal distribution **(D)** None of these

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Consider the following propositions concerned with a certain triangle ABC. **(8)**
p: ABC is isosceles
q: ABC is equilateral
r: ABC is equiangular
Write down the following propositions in words:
(i) $p \wedge (\sim q)$ (ii) $(\sim p) \vee q$
(iii) $p \rightarrow q$ (iv) $q \rightarrow p$
(v) $(\sim r) \rightarrow (\sim q)$ (vi) $p \leftrightarrow (\sim q)$
(vii) $r \rightarrow q$ (viii) $(q \wedge r) \rightarrow p$

b. Is the following arguments valid?
If two sides of a triangle are equal,
then opposite angles are equal
Two sides of a triangle are not equal
$\therefore$ The opposite angles are not equal **(8)**

Q.3 a. If $m = 2$, $n = 5$ and

$$H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Determine the group code $e_H : B^2 \rightarrow B^5$ **(8)**

b. Define group code. Show that (2,5) encoding function $e : B^2 \rightarrow B^5$ defined by $e(00) = 0000,\ e(10) = 10101,\ e(01) = 01110,\ e(11) = 11011$ is a group code. **(8)**

Q.4 a. In a group (G,0), prove that **(8)**
$(a_0 b)^{-1} = b^{-1}{}_0 a^{-1} \quad \forall a, b \in G$

b. What do you mean by addition modulo *m*? Show that the set $G = \{0,1,2,\ldots, m-1\}$ of first m non-negative integers is an Abelian group under the composition addition modulo *m*. **(8)**

Q.5 a. What do you mean by recurrence relation? Solve the following recurrence relation **(8)**
$a_n - 8a_{n-1} + 21a_{n-2} - 18a_{n-3} = 0$

b. Solve the recurrence relation $f_n = f_{n-1} + f_{n-2}, \quad n \geq 2$ with initial conditions $f_0 = 1,\ f_1 = 1$. **(8)**

Q.6 a. Show that $A \cap (B - C) = (A \cap B) - (A \cap C)$ notations are usuals. **(8)**

b. In a city three daily newspapers, X, Y, Z are published. 65% of the people of the city read X, 54% read Y, 45% read Z, 38% read X and Y, 32% read Y and Z, 28% read X and Z, 12% do not read any of the three papers. If 1000000 persons live in the city, find the number of the persons who read all the three newspapers. **(8)**

Q.7 a. Using rules of inference, show that $s \vee r$ is tautologically implied by $p \vee q$, $p \to r, q \to s$ **(8)**

b. Give direct and indirect proof of $p \to q,\ q \to r,\ 7(p \wedge q),\ p \vee r \Rightarrow r$ **(8)**

Q.8 a. If $a = \{1,2,3,4,5,6,7\}$ and a relation R defined as $R = \{(x, y), \|x - y\| = 2\}$. Is R is an equivalence relation? **(8)**

b. If $(L, \leq)$ be a Lattice and $a, b, c \in L$ prove that if $a \leq c, b \leq c$, **(8)**

(i) $a \vee b = a \oplus b \leq c$

(ii) $a \wedge b = a * b \leq c$

Q.9 a. Check whether the function $f(x) = x^2 - 11$ from R to R is one-one onto or both. Justify. **(8)**

b. If $f, g : R \to R$ where $f(x) = ax + b,\ g(x) = 1 - x + x^2$ and $(g_0 f)(x) = 9x^2 - 9x + 3$, then find the values of "a" and "b". **(8)**