ROLL NO. ________________

DipIETE – ET/CS (Current & New Scheme)

Time: 3 Hours **DECEMBER 2018** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. If $n_{C_{10}} = n_{C_8}$ then $n_{C_{16}}$ is equal to

(A) 153	**(B)** 151
(C) 157	**(D)** 155

b. If the 7th and 13th term of a progression is 23 and 41 respectively, then the 21st term is

(A) 65	**(B)** 60
(C) 55	**(D)** 50

c. If $2\sin^2 x = 3\cos x$ then x is

(A) 60^o	**(B)** 50^o
(C) 40^o	**(D)** 30^o

d. The value of $\sqrt{3}cosec\ 20^0 - sec20^0$ is

(A) 4	**(B)** 5
(C) 6	**(D)** 7

e. If the distance between the points (x, –7), (3, –3) is 5 unit, The value of x is

(A) 0	**(B)** 0 or 6
(C) 6	**(D)** None of these

f. The length of the perpendicular from the straight line $x - 2y - 5 = 0$ drawn from the point (-3,-5) is

(A) $\frac{1}{\sqrt{5}}$	**(B)** $\frac{2}{\sqrt{5}}$
(C) $\frac{3}{\sqrt{5}}$	**(D)** $\frac{4}{\sqrt{5}}$

g. The area of the triangle with vertices (-3, 5), (3, -6), (7, 2) is

(A) 41 units	**(B)** 40 units
(C) 46 units	**(D)** 45 units

h. $\left(\frac{dx}{dy}\right)^2 + 5\sqrt[5]{y} = x$ is

(A) linear of degree 2
(B) non-linear of order 1 and degree 2
(C) non-linear of order 1 and degree 6
(D) None of these

i. The function $f(x) = \begin{cases} x\sin\frac{1}{x}, & if\ x \neq 0 \\ 0, & if\ x = 0 \end{cases}$ is

(A) discontinuous at $x = 0$ **(B)** $Lt_{x\to 0}$ f(x) does not exists
(C) continuous at $x = 0$ **(D)** None of these

j. The value of $\int_0^1 \frac{1-x}{1+x}dx$ is

(A) $2log2 - 1$ **(B)** $log2$
(C) $log2 - 1$ **(D)** $2log2$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Using the principle of mathematical induction, prove that $(2n + 7) < (n + 3)^2$ for all values of $n \in N$. **(8)**

b. If $(1 + x)^n = C_0 + C_1x + C_2x^2 + \cdots + C_nx^n$ then show that $C_0^2 + C_1^2 + C_2^2 + C_3^2 + C_4^2 + \cdots + C_n^2 = \frac{(2n)!}{n!n!}$. **(8)**

Q.3 a. If $A + B + C = \pi$, show that $cosA + cosB + cosC = 1 + 4\frac{sinA}{2}\frac{sinB}{2}\frac{sinC}{2}$. **(8)**

b. Prove that $\left(\frac{cosA+cosB}{sinA-sinB}\right)^n + \left(\frac{sinA+sinB}{cosA-cosB}\right)^n = \begin{cases} 2cot^n\left(\frac{A-B}{2}\right), & \text{if } n \text{ is even} \\ 0, & \text{if } n \text{ is odd} \end{cases}$ **(8)**

Q.4 a. If the area of a triangle formed by the straight line L and the coordinate axis is 5 unit where L is perpendicular to the straight line $5x - y = 1$, find the equation of the straight line L. **(8)**

b. The inclination of a straight line passing through the point (4, 5) is 30^0. Find the coordinate of the point lying on that line whose distance from the given point is 3 unit. **(8)**

Q.5 a. Solve the following systems of linear equations by matrix method. $5x - 7y + z = 11, 6x - 8y - z = 5, 3x + 2y - 6z = 7$. **(8)**

b. Given $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ show that $A^2 - 4A - 5I = 0$. Hence find A^{-1}. **(8)**

Q.6 a. Solve: $(x^2 - y^2)dx = 2xydy.$ **(8)**

b. Solve: $(x^2y - 2xy^2)dx = (x^3 - 3x^2y)dy.$ **(8)**

Q.7 a. Evaluate $\int_{\alpha}^{\beta} \sqrt{(x-\alpha)(\beta - x)}dx.$ **(8)**

b. Integrate $\int tan^{-1}(1 + x + x^2)dx.$ **(8)**

Q.8 a. Find the nth derivative of $\frac{1}{x^2 + a^2}$. **(8)**

b. Find the values of a and b such $Lt_{x \to 0} \frac{x(1+acosx) - bsinx}{x^3} = 1.$ **(8)**

Q.9 a. Find the equation of the parabola whose focus is at (-1, 1) and directrix is $x + y + 1 = 0.$ **(8)**

b. If e and e' are the eccentricities of two conjugate hyperbola prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1.$ **(8)**