ROLL NO. ___________________

Code: AE51/AC51/AT51/AE101/AC101/AT101
Subject: ENGINEERING MATHEMATICS - I

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **DECEMBER 2018** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 to Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If $u = f\left(\frac{y}{x}\right)$, then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ **(B)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$

(C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$ **(D)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

b. If every minor of order r of the matrix A is zero, then the rank of matrix A is

(A) greater than r **(B)** equal to r
(C) less than or equal to r **(D)** less than r

c. The order of convergence of Newton-Raphson method is

(A) 2 **(B)** 3
(C) 0 **(D)** None

d. Using Euler's method, solve $\frac{dy}{dx} = \frac{y-2x}{y}$, $y(0) = 1$; *then* $y(0.1) =$

(A) 2.1616 **(B)** 2.1313
(C) 3.1818 **(D)** 1.1818

ROLL NO. ______________________

e. $e^{-x}(c_1 \cos\sqrt{3}\,x + c_2 \sin\sqrt{3}\,x) + c_3 e^{2x}$ is the general solution of the differential equation:

(A) $(D^3+4)\,y=0$ **(B)** $(D^3-8)\,y=0$

(C) $(D^3+8)\,y=0$ **(D)** $(D^3-2D^2+D-2)\,y=0$

f. The value of the integral $\int_0^{\pi} \sin^7\theta\, d\theta$ is equal to

(A) *zero* **(B)** $\frac{32\pi}{35}$

(C) $\frac{32}{35}$ **(D)** 32π

g. The Inverse Z-transform of $\frac{2z^2+3z}{(z+2)(z-4)}$ is

(A) $\frac{1}{6}(-2)^n + \frac{11}{6}(4)^n$ **(B)** $\frac{11}{6}(-2)^n + \frac{1}{6}(4)^n$

(C) $\frac{11}{6}(-2)^n + \frac{11}{6}(4)^n$ **(D)** $\frac{1}{6}(-2)^n + \frac{1}{6}(4)^n$

h. If n is a positive integer the $J_{-n}(x)$ is equal to

(A) $J_n(x)$ **(B)** $(-1)^n J_n(x)$

(C) $(-1)^n J_{-n}(x)$ **(D)** $[-J_n(x)]^n$

i. The solution of the differential equation $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-x/y\right)dy = 0$, is

(A) $x + y e^{y/x} = c$ **(B)** $x + y e^{x/y} = c$

(C) $y e^{y/x} = c$ **(D)** $x e^{y/x} = c$

j. If m and n are two different integers then the value of $\int_{-1}^{1} P_m(x) P_n(x)\, dx$ is

(A) $\frac{2}{2n+1}$ **(B)** 0

(C) $\frac{\pi n}{2n+1}$ **(D)** $n+\frac{\pi}{2}$

ROLL NO. ____________________

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $u = \sin^{-1}\left[\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right]$, then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{-\sin u\cos 2u}{4\cos^3 u}.$$ **(8)**

b. Find the extreme values of $u = x^2y^2-5x^2-8xy-5y^2$. **(8)**

Q.3 a. Change the order of integration of $\int_0^\infty\int_0^\infty e^{-xy}\sin nx\,dx\,dy$, hence show that

$$\int_0^\infty \frac{\sin nx}{x}dx=\frac{\pi}{2}.$$ **(8)**

b. Evaluate $\iiint_R \left(x^2+y^2+z^2\right)dx\,dy\,dz$, where R denotes the region bounded by $x=0, y=0, z=0$ *and* $x+y+z=a, (a>0)$. **(8)**

Q.4 a. Discuss the consistency of the following system of equations:

$$2x+3y+4z=11, x+5y+7z=15, 3x+11y+13z=25.$$

If found consistent , then solve them. **(8)**

b. Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}.$$ **(8)**

Q.5 a. Find the real root of the equation $x^3-2x-5=0$, correct to three decimal places by using Newton-Raphson method. **(8)**

b. Apply Runge-Kutta method of order 4, to find an approximate value of y, *when* $x=0.2$, given that $\dfrac{dy}{dx}=x+y$, $y=1$, *when* $x=0$. **(8)**

Q.6 a. Solve the differential equation $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}+y=\log x\sin(\log x)$. **(8)**

b. Solve the differential equation $\left(D^2-2D+1\right)y=e^x\log x$, by the method of variation of parameters. **(8)**

Q.7 a. Obtain the series solution of $\left(1-x^2\right)y''-xy'+4y=0$. **(8)**

b. Evaluate $\int_0^\infty e^{-ax}x^{m-1}\sin bx\,dx$ in terms of Gamma function. **(8)**

Q.8 a. Prove that $P_n(x)=\frac{1}{n!2^n}\frac{d^n}{dx^n}(x^2-1)^n$ **(8)**

b. Prove that $\frac{d}{dx}[x^{-n}J_n(x)]=-x^{-n}J_{n+1}(x)$. **(8)**

(For current scheme students i.e. AE51/AC51/AT51)

Q.9 a. Solve the differential equation $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$. **(8)**

b. Solve the differential equation $ye^{xy}\,dx+(xe^{xy}+2y)\,dy=0$. **(8)**

(For New scheme students i.e. AE101/AC101/AT101)

Q.9 a. Find the Fourier sine and Fourier cosine transform of $f(x)=e^{-ax}$. **(8)**

b. Find the Fourier series of the function $f(x)=|x|,\ -1<x<1$. **(8)**