ROLL NO. ____________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **DECEMBER 2018** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. Which of the following set is uncountable

(A) $S = \left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots\right\}$ **(B)** $A = \{1,2,4,7,11,16, \ldots\}$

(C) $Z = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$ **(D)** open interval (0,1)

b. $p \rightarrow q$ is equivalent to

(A) $\urcorner p \vee q$ **(B)** $p \vee \urcorner q$

(C) $\urcorner p \wedge q$ **(D)** $\urcorner p \wedge \urcorner q$

c. If $F_0, F_1, F_2, \ldots$ are Fibonacci numbers, then $F_{6=}$

(A)3 **(B)**8

(C)5 **(D)**13

d. Let A be the set of all positive integers and R be the relation on A defined by aRb if and only if $a = b^k$ for some positive integer k. Find which of the following belongs to R?

(A) (3,9) **(B)** (2,5)

(C) (5,2) **(D)** (9,3)

e. The function defined by $f(x) = \sin x$ is one-to-one when its domain is

(A) $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ **(B)** $-\pi \leq x \leq \pi$

(C) $0 \leq x \leq \pi$ **(D)** $0 \leq x \leq \frac{\pi}{2}$

f. In Lattice L, $((a \wedge b) \vee a) \wedge b$ is

(A) $a \wedge b$ **(B)** $a \vee b$

(C) $(a \wedge b) \vee a$ **(D)** $((a \vee b) \wedge a) \vee$

g. $(Z, +)$ has cyclic subgroup of

(A) order 2 **(B)** any order

(C) prime order **(D)** infinite order

h. If 4 x 7 matrix is a generator matrix for a linear code C, then a parity check matrix for C is a

(A) 7x3 matrix **(B)** 7x4 matrix
(C) 4x4 matrix **(D)** 7x7 matrix

i. A finite commutative ring with identity is a field if

(A) R has no zero divisors
(B) R has unique multiplicative identity
(C) The number elements in R is prime
(D) The number of elements in R is a power of a prime

j. $A - \phi$ and $\phi - A$ are respectively

(A) $A, \bar{A}$ **(B)** ϕ, ϕ
(C) A, ϕ **(D)** A, A

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. In how many ways can a sum of 3 or 10 be obtained when two dice are thrown? **(4)**

b. If A={1,2,4,6,8} $B = \{3,5,7\}$ and C={2,4,5,9}. Find $A - C, A \times (B \cap C)$ and $A \Delta B$. **(6)**

c. Suppose A and B are events with P(A)=0.6, P(B)=0.3 and P(A∩ B)=0.2. Find the probability that (i) A does not occur (ii) B does not occur (iii) A or B occurs (iv) Neither A nor B occurs. **(6)**

Q.3 a. Show that $\urcorner P \to (Q \to R) \Leftrightarrow Q \to (P \vee R)$. **(6)**

b. Examine whether the formula $\urcorner((P \to Q) \to ((R \vee P) \to (R \vee Q))$ is Tautology, contradiction or contingency? **(8)**

c. Symbolize the statement “Babu is happy if and only if Babu is not rich”. **(2)**

Q.4 a. Show that $(\forall x)\big(P(x) \vee Q(x)\big) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. **(8)**

b. Prove that $\sqrt{2}$ is not a rational number. . **(6)**

c. Let x and y denote integers. Consider the statement p(x,y) : x+y is even. Write down the following statements in words:

(i) ∀ x , ∃ y p(x,y)
(ii) ∃ x ∀ y p(x,y) **(2)**

Q.5. a. Prove by mathematical induction that, for any positive integer n,

$$1^2 + 3^2 + 5^2 + \cdots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1).$$ **(8)**

b. For integers m and k, the Eulerian numbers $a_{m,k}$ are defined recursively as follows:

$a_{0,0} = 1$,

$a_{m,k} = 0$ for $k \geq m > 0$,

$a_{m,k} = 0$ for $k < 0$,

$a_{m,k} = (m-k)a_{m-1,k-1} + (k+1)a_{m-1,k}$ for $0 \leq k \leq m-1$.

Determine the value of $a_{m,k}$ for $1 \leq m \leq 5$ and $0 \leq k \leq m-1$. **(8)**

Q.6. a. For any non-empty sets A, B, C prove the following results:
(i)$A \times (B \cup C) = (A \times B) \cup (AXC)$ (ii) $A \times (B - C) = (A \times B) - (A \times C)$ **(8)**

b. Let $A = \{1,2,3,4,6\}$ and R be a relation on A defined by aRb if and only if a is a multiple of b. Represent the relation R as a matrix and draw its digraph**.** **(8)**

Q.7. a. Let $A = \{1,2,3,4,5\}$. Define a relation R on $A \times A$ by $(x_1, y_1)R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$. Verify that R is an equivalence relation on $A \times A$. Determine the equivalences classes $[(1,3),(2,4)]$ and $[(1,1)]$. Determine the partition of $A \times A$ induced by R. **(8)**

b. Draw the Hasse diagram for the poset $(\mathcal{P}(S), \subseteq)$, where $S = \{1,2,3,4\}$ and determine whether (P(S), Λ, V) is a lattice or not. **(8)**

Q.8. a. In each of the following cases, determine whether the given set together with the given binary operation is a group or not. If it is a group, indicate if it is abelian; also specify the identity and the inverse of an element:

(i) {-1, 1 } under usual multiplication

(ii) { -1 ,0 ,1} under usual addition

(iii) { 10n/ n ϵ z} under usual addition

(iv) The set of all mxn matrices under matrix addition. **(8)**

b. Let G be the set of all non-zero real numbers and let $a * b = \frac{1}{2}ab$. Show that $(G,*)$ is an abelian group. **(8)**

Q.9 a. An encoding function $E: Z_2^2 \rightarrow Z_2^5$ is given by the generator matrix
$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$. Determine all the code words. Find the associated parity-check matrix H. Use H to decode the received words: 11101,11011. **(8)**

b**.** Prove that the set Z with binary operations ⊕ and ʘ defined by

x ⊕ y = x + y - 1

x ʘ y = x + y - xy

is a commutative ring with unity. Is this ring an integral domain or a field? **(8)**