

DipIETE – ET/CS (Current Scheme)

Time: 3 Hours

December 2016

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. $\lim_{x \rightarrow 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right) =$

- (A) 1 (B) 0
(C) ∞ (D) -1

b. $\int_0^{\pi} \sin^2 \theta \cos^3 \theta d\theta =$

- (A) 0 (B) π
(C) 2/15 (D) 5π

c. The modulus of $-4-3i$ is

- (A) 1 (B) 5
(C) 7 (D) $4+3i$

d. If two vectors $2\vec{i}+3\vec{j}-4\vec{k}$ and $\vec{i}+2\vec{j}+a\vec{k}$ are perpendicular then a is equal to

- (A) 0 (B) -2
(C) 2 (D) 8

e. The solution of $\frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + y = 0$ is

- (A) $Ae^{(-2+\sqrt{3})x} + Be^{(-2-\sqrt{3})x}$ (B) $e^{-2x}(A\cos\sqrt{3}x + B\sin\sqrt{3}x)$
(C) $Ae^{-2x} + Be^{(\sqrt{3})x}$ (D) $e^{-2x}(A\cos\sqrt{3}x - B\sin\sqrt{3}x)$

f. If $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi. \end{cases}$ then $f(0) =$

- (A) $-\pi$ (B) 0
(C) $-\frac{\pi}{2}$ (D) π

- g. A periodic function is given by a function which
 (A) has a period $T=2\pi$ (B) satisfies $f(t+T)=f(t)$
 (C) has a period $T=\pi$ (D) satisfies $f(t+T)=-f(t)$
- h. $\int_0^{\infty} te^{-3t} \sin t \, dt =$
 (A) 0 (B) ∞
 (C) 3 (D) $3/50$
- i. $L^{-1} \left[\frac{1}{s-a} \right] =$
 (A) e^{at} (B) e^{-at}
 (C) $\sin at$ (D) $\cos at$
- j. Which one of the following function is an even function?
 (A) $\sin x$ (B) x^3
 (C) $x \sin x$ (D) $x \cos x$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

- Q.2 a. Verify Lagrange's Mean value theorem for the function
 $f(x) = (x-1)(x-2)(x-3)$ in $(0,4)$ and find the appropriate value of c . (8)
- b. Expand $\log_e x$ in powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to 4 decimal places. (8)
- Q.3 a. Find the entire length of the cardioid $r = a(1 + \cos \theta)$. (8)
- b. Find the surface area of the solid generated by the revolution of the astroid
 $x = a \cos^3 t, y = a \sin^3 t$, about the y-axis. (8)
- Q.4 a. Find the real values of x, y so that $-3 + ix^2y$ and $x^2 + y + 4i$ may represent complex conjugate numbers. (8)
- b. If $2 \cos \theta = x + \frac{1}{x}$, prove that $\frac{x^{2n+1} + 1}{x^{2n-1} + x} = \frac{\cos n\theta}{\cos(n-1)\theta}$. (8)
- Q.5 a. Solve $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 4y = 8(e^{2x} + \sin 2x + x^2)$. (8)
- b. A simple pendulum of length l is oscillating through a small angle θ in a medium in which the resistance is proportional to the velocity. Find the differential equation of its motion. Discuss the motion and find the period of oscillation. (8)

Q.6 a. Expand $f(x) = x \sin x$ as a Fourier series in the interval $0 < x < 2\pi$. (8)

b. Obtain the first three coefficients in the Fourier cosine series for y , where y is given in the following table: (8)

x	0	1	2	3	4	5
y	4	8	15	7	6	2

Q.7 a. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$. (8)

b. Find the Laplace transform of the function (8)

$$f(x) = \begin{cases} \sin \omega t, & 0 < t < \frac{\pi}{\omega} \\ 0, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases}$$

Q.8 a. Apply Convolution theorem to evaluate $L^{-1} \left(\frac{s}{(s^2 + a^2)^2} \right)$. (8)

b. Use transform method to solve $\frac{d^2 x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1, x\left(\frac{\pi}{2}\right) = -1$. (8)

Q.9 a. Constant forces $P = 2I - 5J + K$ and $Q = -I + 2J - K$ act on a particle. Determine the work done when the particle is displaced from A to B, the position vectors of A and B being $4I - 3J - 2K$ and $6I + J - 3K$ respectively. (8)

b. A rigid body is spinning with angular velocity 27 radians per second about an axis parallel to $2I + J - 2K$ passing through the point $I + 3J - K$. Find the velocity of the point of the body whose position vector is $4I + 8J + K$. (8)