

DiplETE – ET/CS (Current & New Scheme)

Time: 3 Hours

December-2016

Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The value of $\lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{\sin x}$ is

- (A) 1 (B) 0
(C) $\frac{1}{2}$ (D) None of these

b. The equation of a circle which passes through the intersection of the lines $3x - 2y = 1$ and $4x + y = 27$ and whose centre is at point $(2, -3)$ is

- (A) $(x + 2)^2 + (y - 3)^2 = 100$ (B) $(x - 2)^2 + (y + 3)^2 = 100$
(C) $(x - 2)^2 + (y + 3)^2 = 109$ (D) $(x + 2)^2 + (y - 3)^2 = 109$

c. The value of $\int_0^1 x e^x dx$ is

- (A) 1 (B) 2
(C) -2 (D) None of these

d. In what ratio is the line joining the points A(4, 4) and B(7,7) is divided by P(-1, 1).

- (A) (7 : 1) (B) (2 : 3)
(C) (5 : 3) (D) (5 : 8)

e. Change of the polar equation of $\theta = \frac{\pi}{4}$ into Cartesian equation is

- (A) $y = 2x$ (B) $x = 4y$
(C) $x = 2y$ (D) $y = x$

- f. If $f(x) = x \cos x - \sin x$, then $f'(x)$ is equal to
 (A) $x \sin x$ (B) $-x \sin x$
 (C) $x \cos x$ (D) $-x \cos x$
- g. If one end of the diameter of the circle $x^2 + y^2 + 4x + 6y - 12 = 0$ is $(1, 1)$ then other end of the diameter is equal to
 (A) $(-5, -7)$ (B) $(-2, -3)$
 (C) $(-3, 5)$ (D) $(2, 7)$
- h. If $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$, then $A^2 - 4A + 7I$ is
 (A) $\begin{bmatrix} 1 & 6 \\ 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
 (C) $\begin{bmatrix} -1 & 8 \\ 0 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- i. If $\cos(A-B) = 3\cos(A+B)$, then $\cot A \cot B$ is:
 (A) -2 (B) 2
 (C) 1 (D) 3
- j. $\int \tan^2 x \, dx$ is equal to
 (A) $\tan x$ (B) $\tan x \cdot \sec x$
 (C) $\tan^2 x$ (D) $\tan x - x$

**Answer any FIVE Questions out of EIGHT Questions.
 Each question carries 16 marks.**

Q.2 a. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, then prove that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ (8)

b. Find the equation of the tangent to the curve $y^2 = 3 - 5x$ parallel to the lines $5x - 4y + 13 = 0$ (8)

Q.3 a. Evaluate $\int_0^{\frac{\pi}{2}} \cos^7 x \, dx$ (8)

b. Evaluate $\int_1^2 \frac{1}{x(1+x^2)} \, dx$ (8)

Q.4 a. Prove that the inverse of a square matrix, if it exists, is unique. (8)

b. Find the adjoint of the matrix A, where

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix} \quad (8)$$

Q.5 a. If $y = \log(x + \sqrt{1+x^2})$, Prove that

$$(1+x^2) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0 \quad (8)$$

b. Solve $\frac{dy}{dx} + y \cdot \sec x = \tan x$ (8)

Q.6 a. Let S_n denote the sum of the first n terms of an A.P. If $S_{2n} = 3S_n$, then prove that $\frac{S_{3n}}{S_n} = 6$ (8)

b. Prove that the coefficient of x^r in the expansion of $(1-4x)^{-1/2}$ is $\frac{(2r)!}{(r!)^2}$ (8)

Q.7 a. Prove that $\cos 80^\circ \cos 60^\circ \cos 40^\circ \cos 20^\circ = \frac{1}{16}$ (8)

b. If $A + B + C = \pi/2$, then prove that $\sin 2A + \sin 2B + \sin 2C = 4 \cos A \cdot \cos B \cdot \cos C$ (8)

Q.8 a. Find the equation of the right bisector of the segment joining A(1, 1) and B(2, 3) (8)

b. Find the distance between the lines $9x + 40y - 20 = 0$ and $9x + 40y + 21 = 0$ (8)

Q.9 a. Find the equation of the circle which passes through the points (3, -2), (-2, 0) and having its centre on the line $2x - y = 3$. (8)

b. Find the focus, vertex, axis, latus-rectum and directrix of the parabola $x^2 + 4x + 2y = 0$ (8)