ROLL NO. ____________________

Code: AE51/AC51/AT51/ AE101/AC101/AT101
Subject: ENGINEERING MATHEMATICS-I

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours **December 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Q2 TO Q8 CAN BE ATTEMPTED BY BOTH CURRENT AND NEW SCHEME STUDENTS.**
- **Q9 HAS BEEN GIVEN INTERNAL OPTIONS FOR CURRENT SCHEME (CODE AE51/AC51/AT51) AND NEW SCHEME (CODE AE101/AC101/AT101) STUDENTS.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. For a 3×3 matrix if 1 , -1 , 0 are the eigen values, then the determinant and transpose of the matrix are

(A) 1, 0 **(B)** 2, 1
(C) 0, 0 **(D)** 0, 1

b. If $f\left(x,y\right)=\frac{1}{x^2}+\frac{1}{xy}+\frac{\log x-\log y}{x^2+y^2}$ then $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=$

(A) -2 f **(B)** 0
(C) 2 f **(D)** Partial derivatives don't exist at (0, 0).

c. The value of $\int_0^4\int_0^{x^2}(x+y)\,dx\,dy$ is

(A) 186.4 **(B)** 176.4
(C) 166.4 **(D)** 156.4

d. For what value of K the equation $x^2Sin(y^K)dx+x^3y^2Cos(y^K)dy=0$ is exact:

(A) K = 0 **(B)** K = 1
(C) K = 2 **(D)** None of these

e. The value of the integral $\int_0^{\frac{\pi}{2}}\sin^3\theta\cos^5\theta d\theta$ is

(A) $\frac{1}{12}$ **(B)** $\frac{1}{48}$
(C) $\frac{1}{24}$ **(D)** The integral does not converge.

f. Let A be a 2×2 matrix with non-zero entries such that $A^2 = I$ then which one of the following is true

(A) $tr(A) = 0$ **(B)** $|A| = 0$

(C) $|A| = 1$ **(D)** None of these

g. The Particular integral of the differential equation $(D^2 - 1)y = a^x$ is:

(A) $\frac{1}{(\log a)^2 - 1}$ **(B)** $\frac{1}{(\log x)^2 - 1} a^x$

(C) $\frac{1}{(\log a)^2} a^x$ **(D)** $\frac{1}{(\log a)^2 - 1} a^x$.

h. Consider the series $x_{n+1} = \frac{x_n}{2} + \frac{9}{8x_n}$, $x_0 = 0.5$ is obtained from the Newton Raphson method. The series converges to

(A) 1.5 **(B)** 1.98

(C) 1.8 **(D)** 2

i. The Fourier Transformation of the exponential signal e^{jw_0t} is

(A) A constant **(B)** A rectangular gate

(C) An impulse **(D)** A series of impulses

j. If P_n is the Legendre function then the value of $\int_{-1}^{1} P_n(x)\,dx$ is

(A) 1 **(B)** 0

(C) 1 if n is even, 0 if n is odd **(D)** The integral doesn't converge

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Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If $u = (1 - 2xy + y^2)^{-\frac{1}{2}}$ Prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2 u^3$. **(8)**

b. If $u = \sin^{-1} \frac{x+y}{\sqrt{x} + \sqrt{y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$. **(8)**

Q.3 a. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{1} \frac{dz\,dy\,dx}{\sqrt{x^2+y^2+z^2}}$. **(8)**

b. Find the area lying inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$. **(8)**

Q.4 a. Reduce the following matrix to the normal form and determine its rank $\begin{bmatrix} 1 & -1 & 3 \\ 3 & 2 & 1 \\ 7 & -2 & 13 \end{bmatrix}$. **(8)**

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b. Find the characteristic equation of the matrix, $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence compute A^{-1}. **(8)**

Q.5 a. Use Newton-Raphson method to find the real root of the equation $x \log_{10} x = 1.2$ correct to three decimal places. **(8)**

b. Using Runge-Kutta method of order 4, solve $\frac{dy}{dx} = \frac{y^2 + x^2}{y^2 - x^2}$ *with* $y(0) = 1$ at $x = 0.2,\ 0.4$. **(8)**

Q.6 a. Solve the differential equation $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65\cos(\log x)$. **(8)**

b. Solve the system of differential equations **(8)**
$(D-1)x + Dy = 2t + 1,$
$(2D+1)x + 2Dy = t.$

Q.7 a. Show that $\iiint_V x^{l-1} y^{m-1} z^{n-1} dxdydz = \frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n)}$. where V is the region $x \ge 0, y \ge 0, z \ge 0$ and $x + y + z \le 1$ **(8)**

b. Solve by Frobenius method $(1 + x^2) y'' + xy' - y = 0$. **(8)**

Q.8 a. Show that $4J_0'''(x) + 3J_0'(x) + J_3(x) = 0$ **(8)**

b. Prove that $P'_{n+1} + P'_n = P_0 + 3P_1 + 5P_2 + ... + (2n+1)P_n$ Where $P_n(x)$ is the legendre's polynomial. **(8)**

Q.9 **(For Current Scheme students i.e. AE51/AC51/AT51)**

a. Solve the differential equation $xy(1 + xy^2)\frac{dy}{dx} = 1$. **(8)**

b. Solve $xdy - ydx = \sqrt{x^2 + y^2}\, dx$ **(8)**

Q.9 **(For New Scheme students i.e. AE101/AC101/AT101)**

a. Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 < x < 2, \\ 0, & x > 2. \end{cases}$ **(8)**

b. Find the z-transform of $a^n \cosh n\theta$. **(8)**