ROLL NO. ___________________

Code: AC65/AC116 **Subject: DISCRETE STRUCTURES**

AMIETE – CS (Current & New Scheme)

Time: 3 Hours **December 2016** **Max. Marks: 100**

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. Four coins are tossed simultaneously. The probability that at least one head and one tail turn up is

(A) 1/16 **(B)** 1/8
(C) 7/8 **(D)** 15/16

b. Let P(S) denotes the powerset of set S. Which of the following is always true?

(A) P(P(S))=P(S) **(B)** P(S) ∩ P(P(S))= {ϕ}
(C) P(S) ∩ S=P(S) **(D)** S is not an element of P(S)

c. Consider the following recurrence relation

$T(1)= 1$
$T(n+1)= T(n) + \text{Floor}(\sqrt{n+1})$ for all $n\geq 1$
The value of $T(m^2)$ for $m\geq 1$ is

(A) $(m/6)\ (21m-39) + 4$
(B) $(m/6)\ (4m^2-3m+5)$
(C) $(m/2)\ (3m^{2.5} -11m+20)-5$
(D) $(m/6)\ (5m^3-34m^2+137m-104)+(5/6)$

d. The propositional statement $(P\rightarrow(Q\vee R))\rightarrow((P\wedge Q)\rightarrow R)$ is

(A) Satisfiable but not valid **(B)** Valid
(C) Contradiction **(D)** None of these

e. Which one of the following is the most appropriate logical formula to represent the statement "Gold and silver ornaments are precious"
The following notations are used:

G(x): x is a gold ornament
S(x): x is a silver ornament
P(x): x is precious

(A) $\forall x\ (P(x) \rightarrow (G(x) \wedge S(x)))$ **(B)** $\forall x\ ((G(x) \wedge S(x)) \rightarrow P(x))$
(C) $\exists x\ ((G(x) \wedge S(x)) \rightarrow P(x))$ **(D)** $\forall x\ ((G(x) \vee S(x)) \rightarrow P(x))$

f. What is the possible number of reflexive relations on a set of 5 elements?
(A) 2^{10} **(B)** 2^{15}
(C) 2^{20} **(D)** 2^{25}

g. Consider the set S={1, ω, ω^2}, where ω, ω^2 are cube roots of unity. If * denotes the multiplication operation, the structure (S, *) forms
(A) A group **(B)** A ring
(C) An integral domain **(D)** A field

h. The inclusion of which of the following sets into
S= {{1,2},{1,2,3}, {1,3,5}, {1,2,4}, {1,2,3,4,5}} is necessary and sufficient to make S a complete lattice under the partial order defined by set containment?
(A) {1} **(B)** {1},{2,3}
(C) {1}, {1,3} **(D)** {1}, {1,3}, {1,2,3,4}, {1,2,3,5}

i. Let f: B→C and g: A→B be two functions. Let h= $f \circ g$. Given that h is an onto function which one of the following is TRUE?
(A) f and g should both be onto functions
(B) f should be onto but g need to be onto
(C) g should be onto but f need not be onto
(D) Both f and g need to be onto

j. How many different non-isomorphic abelian groups of order 4 are there?
(A) 2 **(B)** 3
(C) 4 **(D)** 5

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that S is a valid inference from the premises
P→~Q, Q∨R, ~S→P, ~R **(8)**

b. Without using truth tables, show that Q∨(P∧7Q)∨(7P∧7Q) is a tautology **(4)**

c. Without using truth tables, show that (7P∧(7Q∧R))∨(Q∧R)∨(P∧R) ⇔R **(4)**

Q.3 a. Prove that the intersection of any two subgroups of a group G is again a subgroup of G. **(8)**

b. Prove that every subgroup of a cyclic group is cyclic. **(8)**

Q.4 a. If f : A→B and g : B→C are bijective functions then prove that
$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ **(8)**

b. If A=B=C=R, the set of all real numbers, if f : A→B; g: B→C; $f(a)=2a+1$; $g(b)=b/3$ then find
(i) $f \circ g(-2)$
(ii) $g \circ f(-1)$
(iii) Verify that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$ **(8)**

Q.5 a. Suppose U is a universal set and A, B1, B2, ……, Bn ⊆ U. Prove that A∩(B1 ∪ B2 ∪ …… ∪Bn) =(A∩B1) ∪ (A∩B2) ∪ …….. (A∩Bn) **(8)**

b. Define a Ring. Find all integers k and m for which (z,⊕, Θ) is a Ring under the binary operations
$x \oplus y = x + y - k$, $x \Theta y = x + y - m x y$ **(8)**

Q.6 a. In a distributive lattice, show that (a*b) ⊕ (b*c) ⊕ (c*a) = (a⊕b) * (b⊕c) * (c⊕a) **(4)**

b. If r_1 and r_2 are equivalence relations in set A, prove that $r_1 \cap r_2$ is also an equivalence relation in set A. **(6)**

c. State and prove the distributive inequalities of a lattice. **(6)**

Q.7 a. Prove that (∃x) A(x)→B ⇔(x) (A(x)→B) **(7)**

b. Prove that (x) (P(x)→Q(x)), (x) (R(x)→7Q(x)) => (x) (R(x)→7P(x)) **(5)**

c. Prove that (∃x) M(x) follows logically from the premises (x) H(x)→M(x) and (∃x) H(x) **(4)**

Q.8 a. Prove by mathematical induction that for all n≥1,
1+4+7+………..+ (3n-2)= (n(3n-1))/2 **(8)**

b. Solve the recurrence relation $a_{n+1}-a_n = 3n^2-n$, $n\geq0$, $a_0=3$ **(8)**

Q.9 a. Let $H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Find

(i) the hamming code generated by H
(ii) the minimum distance of the code
(iii) if 001110 is the received word, find the corresponding transmitted code word **(9)**

b. A survey of 500 television watchers produced the following information: 285 watch football games, 195 watch hockey games, 115 watch basketball games, 45 watch football and basketball, 70 watch football and hockey games, 50 watch hockey and basketball games, 50 don't watch any of the three games. How many people watch exactly one of the three games? **(7)**