Code: AE56/AC56/AT56/AE107/AC107/AT107 | ROLL NO. ______________

Subject: ENGINEERING MATHEMATICS - II

AMIETE – ET/CS/IT (Current & New Scheme)

Time: 3 Hours | **December 2016** | Max. Marks: 100

PLEASE WRITE YOUR ROLL NO. AT THE SPACE PROVIDED ON EACH PAGE IMMEDIATELY AFTER RECEIVING THE QUESTION PAPER.

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following : (2×10)

a. Stoke's theorem connects :

(A) A line integral and a surface integral
(B) A surface integral and a volume integral
(C) A line integral and a volume integral
(D) Gradient of a function and its surface integral

b. The missing term in the following table is:

x	0	1	2	3	4
f(x)	1	3	9	-	81

(A) 33 **(B)** 31
(C) 30 **(D)** 27

c. The directional derivative of $\phi = 5x^2y - 5y^2z + 2.5z^2x$ at the point $P(1,1,1)$ in direction of the line $\frac{x-1}{2} = \frac{y-3}{-2} = z$, is

(A) $\frac{11}{3}$ **(B)** $\frac{11}{2}$
(C) $11\frac{2}{3}$ **(D)** $\frac{2}{3}$

d. The partial differential equation by eliminating arbitrary function 'f' from $z = e^y f(x+y)$ is

(A) $\frac{\partial z}{\partial y} = y + z - \frac{\partial z}{\partial x}$ **(B)** $\frac{\partial z}{\partial y} = z + \frac{\partial z}{\partial x}$
(C) $\frac{\partial z}{\partial x} = x - \frac{\partial z}{\partial y}$ **(D)** $\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} = x$

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e. A party of n persons take their seats at random at a round table. The probability that two specified persons always sit together is

(A) $\frac{2}{n}$ (B) $\frac{2}{n-1}$
(C) $\frac{2}{n-2}$ (D) None of these

f. The invariant points of the bilinear transformation $w = \frac{az+b}{cz+d}$

(A) $az^2+(b-c)z-d=0$ (B) $az^2+(b+c)+d=0$
(C) $cz^2+(d-a)z-b=0$ (D) $cz^2+(d+a)z+b=0$

g. If $\vec{R} = xi + yj + zk$ and $\vec{A}$ is a constant vector, then $\nabla(\vec{A}\cdot\vec{R})$ is equal to

(A) $\vec{A}+\vec{R}$ (B) $\vec{A}-\vec{R}$
(C) $\vec{R}$ (D) $\vec{A}$

h. If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displaces a particle in the xy-plane from (0,0) to (1, 4) along a curve $y = 4x^2$, then the work done is

(A) $\frac{5}{104}$ (B) $\frac{104}{5}$
(C) 104 (D) 52

i. Bilinear transformation $\omega = \frac{az+b}{cz+d}$ is conformal if

(A) $ad - bc = 0$ (B) $ad - bc \neq 0$
(C) $ad + bc = 0$ (D) $ad + bc \neq 0$

j. The solution of the partial differential equation $\frac{\partial^2 z}{\partial y^2} = \sin(xy)$ is

(A) $z = -x^2\sin(xy) + yf(x) + g(x)$ (B) $z = -x^2\sin(xy) - xf(x) + g(x)$
(C) $z = -y^2\sin(xy) + yf(x) + g(x)$ (D) $z = x^2\sin(xy) + yf(x) + g(x)$

Answer any FIVE questions out of EIGHT questions.
Each question carries 16 marks.

Q.2 a. Show that every analytic function w = f(z) defines two families of curves, which form an orthogonal system. **(8)**

b. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riemann equations are satisfied thereof. **(8)**

Q.3 a. Find the Taylor's series expansion of a function of the complex variable $f(z)=\dfrac{1}{(z-1)(z-3)}$ about the point, z = 4. Find its region of convergence. **(8)**

b. Evaluate the following integral using Cauchy integral formula $\int_C \dfrac{4-3z}{z(z-1)(z-2)}dz$ where C is the circle $|z|=\dfrac{3}{2}$. **(8)**

Q.4 a. Prove that $div\left\{\dfrac{f(r)\cdot r}{r}\right\}=\dfrac{1}{r^2}\dfrac{d}{dr}\left(r^2 f\right)$ **(8)**

b. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $Z=x^2+y^2-3$ at the point (2, –1,2) **(8)**

Q.5 a. Use Divergence theorem to evaluate $\iint_S [xdydz+ydzdx+zdxdy]$ where S is the portion of the plane $x+2y+3z=6$, which lies in the first octant. **(8)**

b. Apply Green's theorem to evaluate line integral $\int_C [\sin y dx + x(1+\cos y)dy]$ over a circular path C, $x^2+y^2=a^2$. **(8)**

Q.6 a. Find an approximate value of $\log_e 5$ by calculating to 4 decimal places, by Simpson's $\frac{1}{3}$ rule, $\int_0^5 \dfrac{dx}{4x+5}$, by dividing the range into 10 equal parts. **(8)**

b. Using Lagrange's interpolation formula, find the values of y when x = 10, from the following table: **(8)**

x:	5	6	9	11
y:	12	13	14	16

Q.7 a. Apply Charpit method to solve the equation $px+qy=pq$ **(8)**

b. Use method of separation of variables to solve $3\dfrac{\partial U}{\partial x}+2\dfrac{\partial U}{\partial y}=0$, given that $U(x,0)=4e^{-x}$ **(8)**

Q.8 a. From a pack of 52 cards, 6 cards are drawn at random. Find the probability of the following events : **(8)**
(i) Three are red and 3 are black cards
(ii) Three are kings and 3 are queens

b. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03, 0.15 respectively. One of the insured persons meets an accident. What is the probability that he is a scooter driver? **(8)**

Q.9 a. Fit a Poisson distribution to the set of observations: **(8)**

x	0	1	2	3	4
f	122	60	15	2	1

b. In a normal distribution 31 % of the items are under 45 and 8 % over 64. Find the mean and standard deviation of the distribution (given for area 0.19, z = 0.496 and for the area 0.42, z = 1.405) **(8)**