

Time: 3 Hours

DECEMBER 2011

Max. Marks: 100

NOTE: There are 9 Questions in all.

- Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.
- Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.
- The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.
- Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.
- Any required data not explicitly given, may be suitably assumed and stated.

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. The analytic function whose real part is $e^x \cos y$ is

- (A) e^z (B) e^{-z}
 (C) $e^{\bar{z}}$ (D) None of these

b. Bilinear transformation $\omega = \frac{az+b}{cz+d}$ is conformal if

- (A) $ad - bc = 0$ (B) $ad - bc \neq 0$
 (C) $ad + bc = 0$ (D) $ad + bc \neq 0$

c. The value of the integral $\int_C \frac{e^z}{(z+1)^2} dz$, where C is $|z-1|=3$, is equal to

- (A) 0 (B) $2\pi i$
 (C) $2\pi i e$ (D) $\frac{2\pi i}{e}$

d. If \vec{A} and \vec{B} be the vector joining the fixed points (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively to a variable point (x, y, z) , then $\text{grad}(\vec{A} \cdot \vec{B})$ is equal to

- (A) \vec{A} (B) \vec{B}
 (C) $\vec{A} + \vec{B}$ (D) $\vec{A} - \vec{B}$

e. If s be any closed surface enclosing a volume V and $r^2 = x^2 + y^2 + z^2$, then

$$\int_s \nabla r^2 \cdot d\vec{s} \text{ is equal to}$$

- (A) V (B) $2V$
 (C) $4V$ (D) $6V$

f. If the interval of differencing is 1, the value of $\Delta^3 [(1+x)(1+2x)(1+3x)]$ is

- (A) 36 (B) 18
 (C) 9 (D) 6

g. The first term of the series whose second and subsequent terms are 8, 3, 0, -1, 0 is

- (A) 10 (B) 13
 (C) 15 (D) 18

h. The partial differential equation obtained from the function $z = f(x^2 - y^2)$ is

- (A) $xq - yp = 0$ (B) $xq + yp = 0$
 (C) $xypq = 0$ (D) $\frac{xq}{yp} = \text{const.}$

i. If the probabilities of n independent events are $p_1, p_2, p_3, \dots, p_n$, the probability that at least one of the events will happen is

- (A) $p_1 \cdot p_2 \cdot p_3 \dots p_n$
 (B) $1 - p_1 p_2 p_3 p_4 \dots p_n$
 (C) $1 - (1 - p_1)(1 - p_2)(1 - p_3) \dots (1 - p_n)$
 (D) $p_1 + p_2 + p_3 + \dots + p_n$

j. If four coins are tossed, the expectation of the number of heads is

- (A) 1 (B) 2
 (C) 3 (D) 4

**Answer any FIVE Questions out of EIGHT Questions.
 Each question carries 16 marks.**

Q.2 a. Show that the transformation $\omega = \sin z$, maps the families of lines $x = \text{constant}$ and $y = \text{constant}$ into two families of confocal central conics. (8)

b. Determine the analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$. (8)

Q.3 a. State and prove Cauchy's integral formula. (2+6)

b. Find the Laurent's series expansion of $\frac{z^2-6z-1}{(z-1)(z-3)(z+2)}$ in the region $3 < |z+2| < 5$. (8)

Q.4 a. Show that $\text{Div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \text{curl} \vec{A} - \vec{A} \cdot \text{curl} \vec{B}$ (8)

b. Find the values of ℓ and m so that the surfaces $\ell x^2 y + m z^3 = 4$ may cut the surface $5x^2 = 2yz + 9x$ orthogonally at $(1, -1, 2)$. (4+4)

Q.5 a. Apply Green's theorem to evaluate line integral $\int_C [\sin y dx + x(1 + \cos y) dy]$ over a circular path C , $x^2 + y^2 = a^2$. (8)

b. Use Stoke's theorem to evaluate $\int_C [\sin z dx - \cos x dy + \sin y dz]$ where C is a boundary of the rectangle $0 \leq x \leq \pi, 0 \leq y \leq 1, z = 3$. (8)

Q.6 a. Use Charpit's method to solve the equation $pxy + pq + qy = yz$. (8)

b. Using method of separation of variables, solve $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$ given that $u(x, 0) = 4e^{-x}$ (8)

Q.7 a. The following table gives the viscosity of oil as a function of temperature. Use Lagrange's formula to find viscosity of oil at a temperature of 140° (8)

Temp.:	110°	130°	160°	190°
Viscosity:	10.8	8.1	5.5	4.8

b. Find an approximate value of $\log_e 5$ by calculating to 4 decimal places, by Simpson's $\frac{1}{3}$ rule.

$$\int_0^5 \frac{dx}{4x+5}, \text{ dividing the interval into 10 equal parts.} \quad (8)$$

- Q.8** a. A, B and C throw a dice in succession for a prize to be given to the one who throws six first. Compare their chances of winning. (8)
- b. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of an accident is 0.01, 0.03, 0.15 respectively. One of the insured persons meets an accident. What is the probability that he is a scooter driver? (8)

- Q.9** a. Fit a Binomial distribution for the following data and compare the theoretical frequencies with the actual ones: (8)

x:	0	1	2	3	4	5
f(x):	2	14	20	34	22	8

- b. A car-hire firm has two cars which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days
- (i) on which there is no demand,
- (ii) on which a demand is refused. (Assume $e^{-1.5} = 0.2231$) (8)