ROLL NO. ___________________

Code: AE35/AC35/AT35 **Subject: MATHEMATICS-II**

AMIETE – ET/CS/IT (OLD SCHEME)

Time: 3 Hours **DECEMBER 2011** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Please write your Roll No. at the space provided on each page immediately after receiving the Question Paper.**
- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **The answer sheet for the Q.1 will be collected by the invigilator after 45 Minutes of the commencement of the examination.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The complete solution of the $2\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x\partial y}+2\frac{\partial^2 z}{\partial y^2}=0$ is

(A) $z=f_1(y-2x)+f_2(y-\frac{1}{2}x)$ **(B)** $z=f_1(y-2x)+f_2(y+2x)$

(C) $z=f_1(y-\frac{3}{2}x)+f_2(y+\frac{3}{2}x)$ **(D)** $z=f_1(y-5x)+f_2(y+5x)$

b. Eliminating the arbitrary function from $z=f(x^2-y^2)$, the partial differential equation is

(A) $p^2+q^2=0$ **(B)** $p^2-q^2=0$

(C) $yp+xq=0$ **(D)** $x^2+y^2=0$

c. The probability of getting 4 heads in 6 tosses of a fair coin is

(A) $\frac{1}{2}$ **(B)** $\frac{15}{64}$

(C) $-\frac{1}{2}$ **(D)** $-\frac{15}{20}$

d. The mean of the Binomial distribution with n observations and probability of success p, is

(A) $\sqrt{n\,p}$ **(B)** $p\,q$

(C) $n\,p$ **(D)** $\sqrt{p\,q}$

e. The angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point (2,-1,2) is

(A) $\cos^{-1}(\frac{8}{3\sqrt{21}})$ **(B)** $\sin^{-1}(\frac{8}{3\sqrt{21}})$

(C) 90^0 **(D)** 180^0

f. If $\vec{F}=\text{grad}(x^3+y^3+z^3-3xyz)$, the value of $\text{curl}\,\vec{F}$ is

(A) Constant Vector **(B)** Variable Vector

(C) Zero Vector **(D)** 2i + 3j+ 4k

g. The values of a and b for which the surfaces $ax^2-byz=(a+2)x$ and $4x^2y+z^3=4$ cut orthogonally at (1, -1,2) are

(A) $a=\frac{5}{2}, b=1$ **(B)** $a=1, b=\frac{5}{2}$

(C) $a=-1, b=-1$ **(D)** $a=b=0$

h. The value of $\int_0^{2+i}(\bar{z})^2\,dz$ along 2y = x is

(A) $\frac{14}{3}+i\frac{11}{3}$ **(B)** $\frac{7}{2}+i\frac{5}{2}$

(C) $\frac{11}{3}-i\frac{5}{3}$ **(D)** $\frac{10}{3}-i\frac{5}{3}$

i. The residue of $\oint_c \frac{e^z}{(z+1)^2}dz$ at $|z-1|=3$ is

(A) 1 **(B)** $2\pi i$

(C) $-2\pi i$ **(D)** $\frac{2\pi i}{e}$

j. The value of $\int_0^{2\pi}\frac{d\theta}{2+\cos\theta}$ for $|z|=1$ is

(A) $\frac{2\pi}{\sqrt{3}}$ **(B)** $2\pi i$

(C) 1 **(D)** -1

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. If f (z) is analytic in a region R and $|f(z)|$ is a non – zero constant in R, then show that f (z) is constant in R. **(8)**

b. State and prove Cauchy-Riemann equation in polar coordinates. **(8)**

Q.3 a. The probability that a pen manufactured by a MNC will be defective is $\frac{1}{10}$.
If 12 such pens are manufactured, find the probability that
(i) exactly two will be defective
(ii) at least two will be defective
(iii) none will be defective **(8)**

b. A variable X has the probability distribution

x :	-3	6	9
$P(X = x)$:	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

Find E (X) and E (X^2). Hence evaluate E $(2X+1)^2$. **(8)**

Q.4 a. A vector field is given by $\vec{F} = (\sin y)\, i + x(1+\cos y)\, j$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2$, z = 0. **(8)**

b. Using green's theorem, evaluate $\int_c [(y-\sin x)\, dx + \cos x\, dy]$ where c is the plane triangle enclosed by the line $y = 0,\ x = \frac{\pi}{2}$ and $y = \frac{2}{\pi} x$. **(8)**

Q.5 a. Find $\iint_s \vec{F}.\hat{n}\, ds$, where $\vec{F} = (2x+3z)\hat{i} - (xz+y)\hat{j} + (y^2+2z)\hat{k}$ and S is the surface of the sphere having center at (3, -1,2) and radius 3. **(8)**

b. Prove the following identity:
$\text{Curl}(f\ \vec{v}) = (\text{grad}\, f) \times \vec{v} + f\ \text{Curl}\,\vec{v}$ **(8)**

Q.6 a. State and prove Cauchy's Integral theorem. **(8)**

b. Evaluate $\oint_c \frac{3z^2+z}{z^2-1}\, dz$, where C is the circle $|z-1| = 1$. **(8)**

Q.7 a. Expand cos z in a Taylor's series about $z = \frac{\pi}{4}$. **(8)**

b. Solve $(mz - ny)p + (nx - lz)y = ly - mx$. **(8)**

Q.8 a. Use the method of separation of variable to solve the partial differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$; $u(x,0) = 4\,e^{-x}$ **(8)**

b. A tightly stretched string of length ℓ with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \frac{\pi x}{\ell}$. Find the displacement y (x, y). **(8)**

Q.9 a. In a referendum 60% of voters voted in favour. A random sample of 200 voters was selected. What is the probability that in sample
(i) more than 130 voted in favour
(ii) between 105 and 130 inclusive voted in favour
(iii) 120 voted in favour **(8)**

b. Determine the analytic function $f(z)$, where $f(z) = u(x, y) + i\ v(x, y)$, if $v(x, y) = \log(x^2 + y^2) + x - 2y$. **(8)**